Chapter 2

Question 2.1

Note that the scales on the ordinate and abscissa are different. (i) the gradient of each side of the main trench is around 1/8, which corresponds to a slope of around 7°; (ii) the main trench has a width (at the level of the undamaged surface) of ~ 790 µm and a depth of ~ 46.5 µm, which indicates a depth: width aspect ratio of 1: 17.

Question 2.2

(b) Ra = 3.17 µm.

(c) When L_1 = 3 mm, then Ra = 11.6 µm; when L_1 = 6 mm, then Ra = 3.47 µm; when L_1 = 10 mm, then Ra = 3.19 µm. To remove the form successfully requires the cutoff length to be much smaller than any periodic variation in form.

Question 2.3

(a) Assume that yielding occurs in the plane and not in the ball. If the Tresca yield criterion is assumed, then plastic flow will first occur when the maximum shear stress reaches a value of Y/2, where Y is the uniaxial yield stress. For a ball on plane contact, the maximum shear stress (for $\nu = 0.3$) is $0.31 p_o$ at a depth of $0.48a$. From equation 2.20,

$$p_m = \frac{Y}{3 \times 0.31} = 1.61\ GPa$$

Using equations 2.15 and 2.16, the relative radius of curvature and reduced modulus can be calculated to be R = 6 mm and E^* = 115.4 GPa. Equations 2.21 and 2.19 allow the load and contact radius to be calculated as W = 197 N and a = 198 µm. Finally, the depth at which yield first occurs (= 0.48 a) can thus be shown to be 95 µm below the surface.

(b) Equation 2.21 (or 2.32) indicates that reduction in p_o by a factor of two means that the applied load must be reduced by a factor of eight. The maximum load that can be carried is now 197/8 = 24.6 N.

Question 2.4

From equation 2.30,

$$p_o \propto \frac{1}{\sqrt{R}}$$

Using the individual radii of 80 mm and 20 mm and equation 2.15, it can be shown that the current relative radius of curvature *R* is 16 mm, which means that the new relative radius *R* has to be increased to 25 mm. To achieve this requires the radius of the smaller cylinder to be increased to 400/11 = 36.36 mm.

Question 2.5

Ra represents the arithmetic average deviation from the mean line. For a square-wave profile, *Ra* is simply the amplitude *A* of the wave (i.e. half the peak-to-peak height). For a triangular wave, the average deviation is *A*/2, and so for *Ra* = 1 µm, *A* = 2 µm. This is not dependent upon the wavelength of the profile, and thus *Ra* would be the same for triangular profiles with wavelengths of both 10 µm and 100 µm. This observation indicates that three very different surfaces are described by the same value of *Ra*, and thus shows that caution needs to be exercised in assuming that surfaces with the same *Ra* value have any real topographical similarity to each other.

The equation of a triangular wave with wavelength λ is:

$$z = \frac{4Ax}{\lambda} \quad \text{for } 0 \leq x \leq \lambda/4$$

From equation 2.3:

$$Rq^2 = \frac{1}{L}\int_0^L (z(x))^2 \mathrm{d}x = \int_0^{\lambda/4} \frac{16A^2x^2}{\lambda^2}\,\mathrm{d}x = \frac{A^2}{3}$$

Hence

$$Rq = \frac{A}{\sqrt{3}}$$

For $A = 2$ µm, $Rq = 1.155$ µm.

From equation 2.7:

$$Rku = \frac{1}{Rq^4}\frac{1}{L}\int_0^L (z(x))^4 \mathrm{d}x = \frac{9}{A^4}\frac{4}{\lambda}\int_0^{\lambda/4} \frac{4^4A^4x^4}{\lambda^4}\,\mathrm{d}x = \frac{9}{5} = 1.80$$

(n.b. Rku is independent of A and λ)

Whilst analytical solutions are readily generated for simplified profiles such as these, you would need to use a numerical solution for a real surface profile. You may find it instructive to construct a spreadsheet to represent the triangular profile, and to calculate the Rq and Rku values numerically; you should be able to demonstrate equivalence between the analytical and numerical solutions.

Question 2.6

Use Hamilton's equation (equation 2.34) to calculate the value of p_o required for σ_x to be 800 MPa for the three values of μ. The reduced elastic modulus of the contact E^* is 167 GPa (from equation 2.16) and the relative radius R is 10 mm (from equation 2.15). Values of W are then derived from Eq. 2.21 as follows: (a) $W = 9.7$ N; (b) $W = 286$ N; (c) $W = 2630$ N.

These results show that lubrication of the contact, by reducing the coefficient of friction, generally has a very strong effect of increasing the load-carrying capacity. (However, as we shall see later in Section 5.10.5, some lubricants interact adversely with some ceramics, reducing the fracture toughness and causing a reduction in the fracture stress).