Solutions for Selected Exercises

in

Statistical Thermodynamics and Microscale Thermophysics

Van P. Carey

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1.5

$$\varepsilon = \frac{\hbar^2}{2I}\ell(\ell+1) \qquad \ell \geq |m| \; , \quad m = \pm 1, \pm 2, \ldots$$

$m = \pm 1 \rightarrow \ell = 1, 2, 3, \ldots$

$m = \pm 2 \rightarrow \ell = 2, 3, 4, \ldots$

ℓ	$\ell(\ell+1)$
1	2
2	6
3	12

$$\text{lowest} \rightarrow \varepsilon_0 = \frac{(6.63\times 10^{-34}/2\pi)}{2(6.69\times 10^{-46}} \cdot 2 = \underline{\underline{1.66\times 10^{-23}\ J}}$$

similarly, the next two levels are

$$\varepsilon_1 = \underline{\underline{4.99\times 10^{-23}\ J}} \quad , \quad \varepsilon_2 = \underline{\underline{9.99\times 10^{-23}\ J}}$$

1.7

From eqn's (1.59) and (1.60)

$$H_0(\alpha^{1/2}x) = 1 \; , \quad \psi_0(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} \exp\left\{-\frac{\alpha x^2}{2}\right\}$$

$$\wp(x)\,dx = \psi_0^* \psi_0 \, dx = \left(\frac{\alpha}{\pi}\right)^{1/2} e^{-\alpha x^2} dx \; , \quad \alpha = \frac{\sqrt{km}}{\hbar}$$

(a) The energy of the oscillator in the ground state is $(\hbar/2)(k/m)^{1/2}$. The greatest displacement it could have classically is the displacement where all its energy is potential energy.

$$\rightarrow \frac{1}{2} k x_{max,cl}^2 = \left(\frac{\hbar}{2}\right)\left(\frac{k}{m}\right)^{1/2}$$

$$\rightarrow x_{max,cl} = \left(\frac{\hbar}{k}\right)^{1/2}\left(\frac{k}{m}\right)^{1/4} = \left(\frac{\hbar^2}{km}\right)^{1/4} = \underline{\underline{\alpha^{-1/2}}}$$

(b) $\wp_c$ = probability displacement will exceed $x_{max,cl}$

$$\wp_c = \int_{-\infty}^{-\alpha_c^{-1/2}} \wp(x)\,dx + \int_{\alpha^{-1/2}}^{\infty} \wp(x)\,dx = 2\int_{\alpha^{-1/2}}^{\infty} \wp(x)\,dx$$

$$= 2\int_{\alpha^{-1/2}}^{\infty} \left(\frac{\alpha}{\pi}\right)^{1/2} e^{-\alpha x^2} dx$$

1.7 (Cont'd)

let $\eta = \sqrt{\alpha}\, x \quad \rightarrow \quad dx = d\eta / \sqrt{\alpha}$

$$P_c = \frac{2}{\sqrt{\pi}} \int_1^{\infty} e^{-\eta^2} d\eta = \operatorname{erfc}(1) = \underline{\underline{0.16}}$$

1.10

$$\varepsilon = \frac{3}{2} k_B T = \frac{3}{2}(1.38\times10^{-23})(280) = 5.8\times10^{-21}\ J$$

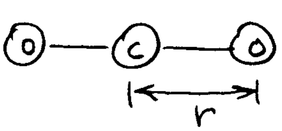
$$\delta\varepsilon = 0.01\,\varepsilon = 5.8\times10^{-23}\ J$$

$$g = \frac{\pi}{4}\left[\frac{8 m_{He} V^{2/3}}{h^2}\right]^{3/2} \varepsilon^{1/2}\,\delta\varepsilon$$

$$= \frac{\pi}{4}\left[\frac{8(6.64\times10^{-27})(0.1)^{2/3}}{(6.63\times10^{-34})^2}\right]^{3/2}(5.8\times10^{-21})^{1/2}(5.8\times10^{-23})$$

$$g = \underline{\underline{1.46\times10^{28}}}$$

1.12



$r = 2.77\ \text{Å}$

$m_c = 1.99\times10^{-26}\ kg \qquad m_o = 2.66\times10^{-26}\ kg$

$$I = \frac{m_o^2 (2r)^2}{2 m_o} = 2 m_o r^2 = 2(2.66\times10^{-26})(2.77\times10^{-10})^2$$

$$I = 4.08\times10^{-45}\ kg\,m^2$$

rotational energy $= k_B T = 1.38\times10^{-23}(290) = 4.0\times10^{-21}\ J$

$$\varepsilon = \frac{\hbar^2}{2I}\,\ell(\ell+1)$$

$$\ell(\ell+1) = \frac{2I\varepsilon}{\hbar^2} = \frac{2(4.08\times10^{-45})(4.0\times10^{-21})}{(1.055\times10^{-34})^2} = 2933$$