Chapter 2

Section 2.7 REVIEW

A short quiz is given in this section. The answers to these questions are given below.

(1) False. The correct answer is $P(E_1 \cup E\ ) = P(E_1) + P(E_2) - P(E_1)P(E_2)$

(2) True. $P(E_1\ E_2) = P(E_1 | E_2)\ P(E_2) = 0$. Thus, $P(E_1 | E_2) = 0$.

(3) False. Union of events is acceptable; union of probability numbers is not possible.

(4) True.

(5) False. The correct answer is $P(E_1 | E_2) = \dfrac{P(E_1 E_2)}{P(E_2)}$.

(6) False. The correct answer is $P(\overline{E}_1|\overline{E}_2) = 1 - P(E_1|\overline{E}_2)$.

(7) False. The question will be correct if E_1 and E_2 are statistically independent.

(8) False. The correct answer is $P(\overline{E}_1|E_2 \cup E_3) = 1 - P(E_1|E_2 \cup E_3)$

(9) True.

(10) True.

Section 2.9 Problems

Problem 2.1

The total number of sample points in the sample space is $3^2 = 9$. They are:

	Car 1	Car 2
1	E	E
2	E	G
3	E	B
4	G	E
5	G	G
6	G	B
7	B	E
8	B	G
9	B	B

For 5 cars, each with 3 possible operating conditions, the total number of sample points is $3^5 =$ 243.

Problem 2.2

Without considering the ordering of lots A and B, the lot can be subdivided in the following different ways

Lot A	Lot B
4	0
3	4
2	8
1	12
0	16

Problem 2.3

(a) The Venn diagram for the sample space is shown in Figure S2.3.

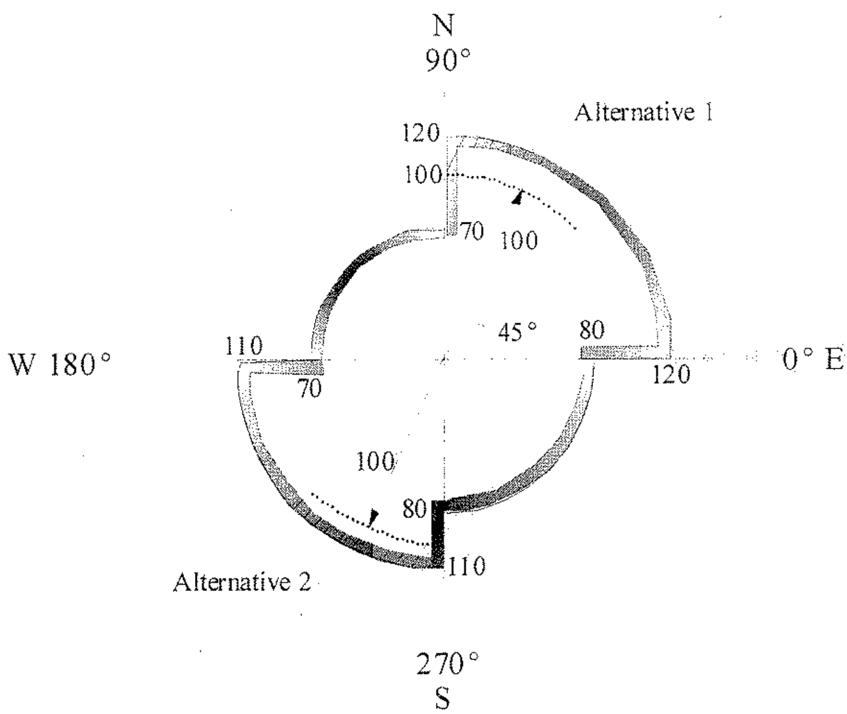


Figure S2.3

(b) In Part (a), the question was intended to indicate that the wind is blowing towards the specified directions. Then the solution is Alternative 1. If the problem is misinterpreted as the wind blowing away from these directions, then the solution is Alternative 2. Both alternatives are shown on the Venn diagram.

Problem 2.4

(a) The sample space consists of 12 $(2 \times 2 \times 3)$ sample points. They are:

	Foundation	Superstructure	Interior
1	G	G	E
2	G	G	G
3	G	G	B
4	G	B	E
5	G	B	G
6	G	B	B
7	B	G	E
8	B	G	G
9	B	G	B
10	B	B	E
11	B	B	G
12	B	B	B

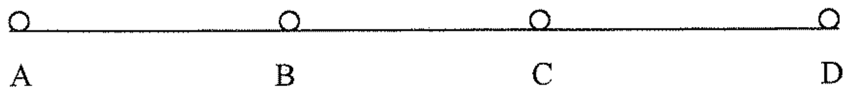
(b) Sample points that make the property marketable are (GGE) and (GGG).

Problem 2.5

(a) The sample space consists of 8 $(2 \times 2 \times 2)$ sample points. They are:

	A	B	C	D
1	0	0	0	0
2	0	0	0	3
3	0	0	2	0
4	0	0	2	3
5	0	1	0	0
6	0	1	0	3
7	0	1	2	0
8	0	1	2	3

(b) If the foundations are arranged in a row as in a typical bridge shown below, the sample points with differential settlement of 2 cm or more are (0,0,0,3), (0,0,2,0), (0,0,2,3), (0,1,0,3), and (0,1,2,0).



However, if the foundations are arranged in a square or rectangle as in a typical building shown below, the sample points with differential settlement of 2 cm or more are (0,0,0,3), (0,0,2,0), (0,0,2,3), (0,1,0,3), (0,1,2,0), and (0,1,2,3).

Problem 2.6

Foundation	Superstructure	Plumbing	Electrical	Painting	Total Completion Time
3	10	1			14
3	10	2			15
3	10		2		15
3	10		3		16
3	10			1	14
3	12	1			16
3	12	2			17
3	12		2		17
3	12		3		18
3	12			1	16
4	10	1			15
4	10	2			16
4	10		2		16
4	10		3		17
4	10			1	15
4	12	1			17
4	12	2			18
4	12		2		18
4	12		3		19
4	12			1	17

The possible completion times are 14, 15, 16, 17, 18, and 19 weeks.

Problem 2.7

Sample Points	Flood Level at A	Flood Level at B	Flood Level at C = 0.5A + 0.25B
1	0	0	0.00
2	0	1	0.25
3	0	2	0.50
4	0	3	0.75
5	1	0	0.50
6	1	1	0.75
7	1	2	1.00
8	1	3	1.25
9	2	0	1.00
10	2	1	1.25
11	2	2	1.50
12	2	3	1.75
13	3	0	1.50
14	3	1	1.75
15	3	2	2.00
16	3	3	2.25

There are 16 sample points and they are equally likely, i.e., the probability of each sample point is 1/16. There is only one sample point indicating that the flood level of river C will exceed 2 ft. Thus, P(Flood level of river C will exceed 2 ft) = 1/16 = 0.0625.

Problem 2.8

(a) $P(0.4 \le D_r) = \dfrac{48}{50} = 0.96$

(b) $P(D_r > 0.8 \mid N > 15) = \dfrac{P(D_r > 0.8 \cap N > 15)}{P(N > 15)} = \dfrac{11}{38} = 0.289$

(c) $P(0.4 < D_r \le 0.8 \mid 5 < N \le 25) = \dfrac{P(0.4 < D_r \le 0.8 \cap 5 < N \le 25)}{P(5 < N \le 25)} = \dfrac{35}{40} = 0.875$

(d) $P(D_r > 0.4 \text{ and } N \le 15) = \dfrac{10}{50} = 0.2$

(e) $P(D_r \le 0.8 \text{ and } N > 25) = \dfrac{2}{50} = 0.04$

Problem 2.9

Let W and E denote the occurrence of high wind and moderate earthquake in a single minute, respectively. The information given in the problem can be summarized as

$P(W) = 10^{-5}$ and $P(E) = 10^{-8}$

(a) $P(WE) = P(W) \times P(E) = 10^{-5} \times 10^{-8} = 10^{-13}$.

Since the joint occurrence of W and E in a single minute is very small compared with the individual occurrence of W and E, the building code is quite reasonable.

(b) P(no E in a minute) = $P(\overline{E}) = (1 - 10^{-8})$

P(no E in a day) = P(no E in 1440 minutes) = $(1 - 10^{-8})^{1440} = 0.9999856$

P(no E in a year) = $(1 - 10^{-8})^{60 \times 24 \times 365} = 0.994758$

P(no E in 50 years) = $(1 - 10^{-8})^{60 \times 24 \times 365 \times 50} = 0.768896$

Problem 2.10

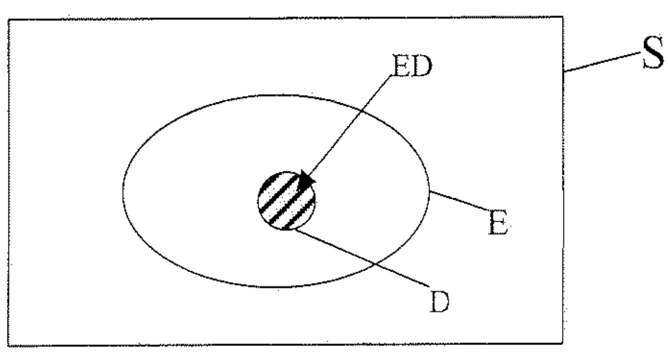
Let E and D denote the failure of electricity and diesel generator to supply power to the operating room. The information given in the problem can be summarized as

$P(E) = 0.001$, $P(D \mid E) = 0.01$

(a) P(no power in the operating room) = $P(ED) = P(D \mid E) \times P(E) = 0.01 \times 0.001 = 10^{-5}$.

(b) If there is no electrical power failure, both electricity and diesel generator are in good operating condition. See the Venn diagram below. Thus,

$$P(\overline{E} \mid \overline{E} \cup \overline{D}) = \frac{P[\overline{E} \cap (\overline{E} \cup \overline{D})]}{P(\overline{E} \cup \overline{D})} = \frac{P(\overline{E})}{P(\overline{E} \cup \overline{D})} = \frac{1 - P(E)}{1 - P(ED)} = \frac{1 - 0.001}{1 - 10^{-5}} = 0.99901$$



Problem 2.11

The information given in the problem can be summarized as

P(I) = 0.1, P(V) = 0.05, and P(V | I) = 0.3.

(a) P(accident) = P(I ∪ V) = P(I) + P(V) – P(IV) = P(I) + P(V) – P(V | I) × P(I)

= 0.1 + 0.05 – 0.3 × 0.1 = 0.12.

(b) P(accident) = P(I ∪ V) = P(I) + P(V) – P(I) × P(V) = 0.1 + 0.05 – 0.1 × 0.05 = 0.145.

Problem 2.12

Let M and S denote the events of failure of a concrete beam due to excessive bending moment and excessive shear force, respectively, and C denotes the appearance of small diagonal cracks. The information given in the problem can be summarized as

$P(M) = 0.95$, $P(S) = 0.05$, $P(C \mid S) = 0.8$, and $P(C \mid M) = 0.1$

$$P(C) = P(C|S)P(S) + P(C|M)P(M) = 0.8 \times 0.05 + 0.10 \times 0.95 = 0.135$$

Thus, $$P(S \mid C) = \frac{P(C|S)P(S)}{P(C)} = \frac{0.8 \times 0.05}{0.135} = 0.296$$

$$P(M|C) = \frac{P(C|M)P(M)}{P(C)} = \frac{0.1 \times 0.95}{0.135} = 0.704$$

Since the probability of shear failure is less than the probability of bending failure, the beam should not be replaced.

Problem 2.13

Let M and V denote the events of failure of the beam in bending and shear, respectively. The information given in the problem can be summarized as

P(M | A) = 0.01, P(M | B) = 0.02, P(S | A) = P(S | B) = 0.001, P(M | S) = 0.9, P(A) = 0.3, and P(B) = 0.7.

Since A and B are mutually exclusive and collectively exhaustive events, using the theorem of total probability (Equation 2.22), we can show that

P(M) = P(M | A) P(A) + P(M | B) P(B) = 0.01 × 0.3 + 0.02 × 0.7 = 0.017

P(S) = P(S | A) P(A) + P(S | B) P(B) = 0.001 × 0.3 + 0.001 × 0.7 = 0.001

P(failure of the beam) = $P(M \cup S) = P(M) + P(S) - P(MS) = P(M) + P(S) - P(M|S)\,P(S)$

$= 0.017 + 0.001 - 0.9 \times 0.001 = 0.0171.$

Problem 2.14

The information given in the problem can be summarized as

$P(E_1) = 0.002,\ P(E_2) = 0.001,\ P(E_3) = 0.001,\ P(E_1|E_2) = P(E_1|E_3) = P(E_1|E_2E_3) = 0.0,$ and $P(E_2|E_3) = 1.0$

P(Failure of the joint) =

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1E_2) - P(E_2E_3) - P(E_3E_1) + P(E_1E_2E_3)$$

$$= 0.002 + 0.001 + 0.001 - 0.0 - 1.0 \times 0.001 - 0.0 + 0.0 = 0.003.$$

Problem 2.15

The information given in the problem can be summarized as

$P(E_1) = 0.10,\ P(E_2) = 0.05,\ P(E_3) = 0.01,\ P(E_2|E_1) = 0.8,\ P(E_3|E_2) = 0.9,\ P(E_3|E_1) = 0.5$, and $P(E_3|E_1E_2) = 0.9.$

(a) Let E_n and E_s denote that the highway will need major repair in the north and south directions, respectively. Then,

$$P(E_n) = P(E_s) = P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1E_2) - P(E_2E_3) - P(E_3E_1) + P(E_1E_2E_3)$$

$$= P(E_1) + P(E_2) + P(E_3) - P(E_2|E_1)\,P(E_1) - P(E_3|E_2)\,P(E_2) - P(E_3|E_1)\,P(E_1)$$

$$+ P(E_3|E_1E_2)\,P(E_2|E_1)\,P(E_1)$$

$$= 0.10 + 0.05 + 0.01 - 0.8 \times 0.1 - 0.9 \times 0.05 - 0.5 \times 0.10 + 0.9 \times 0.8 \times 0.1 = 0.057$$

(b) P(the highway will need major repair) = $P(E_n \cup E_s) = P(E_n) + P(E_s) - P(E_n)P(E_s)$

$$= 0.057 + 0.057 - 0.057 \times 0.057 = 0.11075$$

Problem 2.16

Let E_i denote the failure of the ith pile. Then, $P(E_i) = 10^{-4}$ or $P(\overline{E}_i) = 1 - 10^{-4}$

(a) P(none of the 4 piles will fail under a column) = $P(\overline{E}_1\overline{E}_2\overline{E}_3\overline{E}_4) = (1 - 10^{-4})^4 = 0.9996$

(b) P(none of the 20 columns will suffer damage) = $(0.9996)^{20} = 0.9920.$

Problem 2.17

Let F and E denote the events of a damaging fire and a strong earthquake in a year. The information given in the problem can be summarized as

$P(F) = 0.03$, $P(E) = 0.001$, and $P(F \mid E) = 0.3$

(a) P(Fire due to a strong earthquake in a year) = $P(F_e) = P(F \mid E)\ P(E) = 0.3 \times 0.001 = 0.0003$

(b) P(The subdivision will have fire in a year) = $P(F_e \cup F) = P(F_e) + P(F) - P(F_eF)$

$= P(F_e) + P(F) - P(F_e)\ P(F) = 0.0003 + 0.03 - 0.0003 \times 0.03 = 0.03029$

(c) P(No fire in the subdivision in a year) = $(1.0 - 0.03029) = 0.96971$

P(No fire in the subdivision in 10 years) = $(1.0 - 0.03029)^{10} = 0.73522$.

Problem 2.18

Let F denote the event of failure of the building during an earthquake. The information given in the problem can be summarized as

$P(F \mid L) = 0.01$, $P(F \mid M) = 0.2$, and $P(F \mid H) = 0.70$

(a) $$P(L) = \frac{0.5}{0.5 + 0.05 + 0.001} = \frac{0.5}{0.551} = 0.90744$$

$$P(M) = \frac{0.05}{0.551} = 0.090744$$

$$P(H) = \frac{0.001}{0.551} = 0.001816$$

(b) Using the theorem of total probability (Equation 2.22), we can show that

$$P(F) = P(F|L)P(L) + P(F|M)P(M) + P(F|H)P(H)$$

$$= 0.01 \times 0.90744 + 0.2 \times 0.090744 + 0.7 \times 0.001816 = 0.0285.$$

(c) P(no failure in an earthquake) = $P(\overline{F}) = 1 - 0.0285 = 0.9715$

P(no failure in 2 earthquakes) = $(0.9715)^2 = 0.944$

P(no failure in 100 earthquakes) = $(0.9715)^{100} = 0.0555$.

Problem 2.19

The information given in the problem can be summarized as

$$P(A+|AC)=1.0, P(A^+|A\overline{C})=0.7, P(A^+|\overline{A}C)=0.5, and \quad P(A^+|\overline{A}\,\overline{C})=0.0$$

Also, $P(C|A)=0.90, P(A)=0.95$, and $P(C)=0.90$

(a) Using the theorem of total probability (Equation 2.22), we can show that

$$P(A^+)=P(A^+|AC)P(AC)+P(A^+|A\overline{C})P(A\overline{C})+P(A^+|\overline{A}C)P(\overline{A}C)+P(A^+|\overline{A}\,\overline{C})P(\overline{A}\,\overline{C})$$

$$P(AC)=P(C|A)P(A)=0.90\times 0.95=0.855$$

$$P(A|C)=\frac{P(AC)}{P(C)}=\frac{0.855}{0.90}=0.95$$

$$P(A\overline{C})=P(\overline{C}|A)P(A)=(1-0.90)\times 0.95=0.095$$

$$P(\overline{A}C)=P(\overline{A}|C)P(C)=[1-P(A|C)]P(C)=(1-0.95)\times 0.90=0.045$$

$$P(\overline{A}\,\overline{C})=1-P(AC)-P(A\overline{C})-P(\overline{A}C)=1-0.855-0.095-0.045=0.005$$

Thus, $P(A^+)=1\times 0.855+0.7\times 0.095+0.5\times 0.045+0.0=0.944$

(b) Using Bayes' theorem (Equation 2.23 or 2.24), we can show that

$$P(AC|A^+)=\frac{P(A^+|AC)P(AC)}{P(A^+)}=\frac{1.0\times 0.855}{0.944}=0.906.$$

Problem 2.20

Let E define the event of excessive bacteria in the water supply. The information given in the problem can be summarized as

$$P(A)=\frac{1}{1+1+3}=0.2, \quad P(B)=\frac{1}{5}=0.2, \text{ and } P(C)=\frac{3}{5}=0.6$$

Also, $P(E|A)=0.05$, $P(E|B)=0.1$, and $P(E|C)=0.02$

(a) Using the theorem of total probability (Equation 2.22), we can show that

$$P(E)=P(E|A)P(A)+P(E|B)P(B)+P(E|C)P(C)$$

$$= 0.05 \times 0.2 + 0.1 \times 0.2 + 0.02 \times 0.6 = 0.01 + 0.02 + 0.012 = 0.042$$

(b) Using Bayes' theorem (Equation 2.23), we can show that

$$P(A|\overline{E})=\frac{P(\overline{E}|A)P(A)}{P(\overline{E})}=\frac{[1-P(E|A)]P(A)}{1-P(E)}=\frac{(1-0.05)\times 0.2}{(1-0.042)}=0.198.$$