Chapter 2 – Problem Solutions

2.2.1

$P = \gamma \cdot h$; where $\gamma = (1.03)(9810\ N/m^3) = 1.01 x 10^4\ N/m^3$

(using the specific weight of water at standard conditions since water gets very cold at great depths)

$P = \gamma \cdot h = (1.01 x 10^4\ N/m^3)(730\ m)$

$P = 7.37 x 10^6\ N/m^2 = 1,070\ psi$

The pressure given is gage pressure. To get absolute pressure, atmospheric pressure must be added.

2.2.2

a) The force exerted on the tank bottom is equal to the weight of the water body.

$F = W = m \cdot g = [\rho \cdot (Vol)]\ (g)$

$F = [1.94\ slugs/ft^3\ (\pi \cdot (5\ ft)^2 \cdot 3\ ft)]\ (32.2\ ft/sec^2)$

$F = 1.47 \times 10^4$ lbs

(Note: 1 slug = 1 $lb \cdot sec^2/ft$)

b) The force exerted on the tank bottom is equal to the pressure on the bottom times the area of the bottom.

$P = \gamma \cdot h = (62.3\ lb/ft^3)(3\ ft) = 187\ \ lb/ft^2$

$F = P \cdot A = (187\ \ lb/ft^2)(\ \pi \cdot (5\ ft)^2)$

$F = 1.47 \times 10^4$ lbs

2.2.3

γ_{water} at 30°C = 9.77 kN/m^3

P_{vapor} at 30°C = 4.24 kN/m^2

$P_{atm} = P_{column} + P_{vapor}$

$P_{atm} = (9.8\ m)(9.77\ kN/m^3) + (4.24\ kN/m^2)$

$P_{atm} = 95.7\ kN/m^2 + 4.24\ kN/m^2 = 99.9\ kN/m^2$

2.2.3 (cont.)

The percentage error if the direct reading is used and the vapor pressure is ignored is:

$Error = (P_{atm} - P_{column})/(P_{atm})$

$Error = (99.9\ kN/m^2 – 95.7\ kN/m^2)/(99.9\ kN/m^2)$

Error = 0.0420 = **4.20%**

2.2.4

The atm. pressure found in problem 2.2.3 is 99.9 kN/m^2

$P_{atm} = (\gamma_{Hg})(h)$

$h = P_{atm}/\gamma_{Hg} = (99.9\ kN/m^2)\ /\ [(13.6)(\ 9.77\ kN/m^3)]$

h = 0.752 m = 75.2 cm = 2.47 ft

2.2.5

The force exerted on the tank bottom is equal to the pressure on the bottom times the area of the bottom.

$P = \gamma \cdot h = (9.79\ kN/m^3)(6\ m) = 58.7\ kN/m^2$

$F = P \cdot A = (58.7\ kN/m^2)(36\ m^2)$

F = 2,110 N

The force exerted on the sides of the tank may be found in like manner (pressure times the area). However, the pressure is not uniform on the tank sides since $P = \gamma \cdot h$. Therefore, the average pressure is required. Since the pressure is a linear relationship, the average pressure occurs at half the depth. Now,

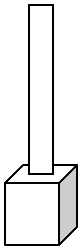
$P_{avg} = \gamma \cdot h_{avg} = (9.79\ kN/m^3)(3\ m) = 29.4\ kN/m^2$

$F = P_{avg} \cdot A = (29.4\ kN/m^2)(36\ m^2)$

F = 1,060 N

Obviously, the force on the bottom is greater than the force on the sides by a factor of two.

2.2.6



$W_{total} = (Vol_{container})(\gamma_{water}) + (Vol_{pipe})(\gamma_{water})$

$W_{total} = (3\ ft)^3(62.3\ lb/ft^3) + [(\pi)(0.50\ ft)^2(30\ ft)](62.3\ lb/ft^3)$

$\mathbf{W_{total}} = 1680\ lb + 1470\ lb = \mathbf{3{,}150\ lb}$

$P_{bottom} = \gamma h = (62.3\ lb/ft^3)(33\ ft) = 2060\ lb/ft^2$

$\mathbf{F_{bottom}} = (2060\ lb/ft^2)(9\ ft^2) = \mathbf{18{,}500\ lb}$

Note: The weight of the water is not equal to the force on the bottom. Why? (Hint: Draw a free body diagram of the 3 ft x 3 ft x 3 ft water body labeling all forces (vertical) acting on it. Don't forget the pressure from the container top. Now, to determine the side force:

$P_{avg} = \gamma \cdot h_{avg} = (62.3\ lb/ft^3)(31.5\ ft) = 1960\ lb/ft^2$

$F = P_{avg} \cdot A = (1960\ lb/ft^2)(9\ ft^2)$

F = 17,600 lb

2.2.7

$P_{bottom} = P_{gage} + (\gamma_{liquid})(1.4\ m)$; and

$\gamma_{liquid} = (SG)(\gamma_{water}) = (0.80)(9790\ N/m^3) = 7830\ N/m^3$

$\therefore P_{bottom} = 4.50 x 10^4\ N/m^2 + (7830\ N/m^3)(1.4\ m)$

$\mathbf{P_{bottom} = 5.60 \times 10^4\ N/m^2}$

The pressure at the bottom of the liquid column can be determined two different ways which must be equal. Hence,

$(h)(\gamma_{liquid}) = P_{gage} + (\gamma_{liquid})(1\ m)$

$h = (P_{gage})/(\gamma_{liquid}) + 1m$

$\mathbf{h} = (4.50 x 10^4\ N/m^2)/7830\ N/m^3 + 1\ m = \mathbf{6.75\ m}$

2.2.8

$\gamma_{seawater} = (SG)(\gamma_{water}) = (1.03)(9790\ N/m^3)$

$\gamma_{seawater} = 1.01 \times 10^4\ N/m^3$

$P_{tank} = (\gamma_{water})(\Delta h) = (1.01 \times 10^4\ N/m^3)(6\ m)$

$\mathbf{P_{tank} = 6.06 \times 10^4\ N/m^2}$ **(Pascals) = 8.79 psi**

2.2.9

$\gamma_{oil} = (SG)(\gamma_{water}) = (0.85)(62.3\ lb/ft^3) = 53.0\ lb/ft^3$

$P_{10ft} = P_{air} + (\gamma_{oil})(10\ ft)$

$P_{air} = P_{10ft} - (\gamma_{oil})(10\ ft)$

$P_{air} = 23.7\ psi\ (144\ in^2/ft^2) - (53.0\ lb/ft^3)(10\ ft)$

$\mathbf{P_{air} = 2.88 \times 10^4\ lb/ft^2}$ **(20.0 psi); Gage pressure**

$P_{abs} = P_{gage} + P_{atm} = 20.0\ psi + 14.7\ psi$

$\mathbf{P_{abs} =}$ **34.7 psi** $\mathbf{(5.00 \times 10^4\ lb/ft^2)}$**; Absolute pressure**

2.2.10

The mechanical advantage in the lever increases the input force delivered to the hydraulic jack. Thus,

$F_{input} = (9)(50\ N) = 450\ N$

The pressure developed in the system is:

$P_{input} = F/A = (450\ N)/(25\ cm^2) = 18\ N/cm^2$

$\mathbf{P_{input} = 180\ kN/m^2}$

From Pascal's law, the pressure at the input piston should equal the pressure at the two output pistons.

$\therefore$ The force exerted on each output piston is:

$P_{input} = P_{output}$ equates to: $18\ N/cm^2 = F_{output}/250\ cm^2$

$F_{output} = (18\ N/cm^2)(250\ cm^2)$

$\mathbf{F_{output} = 4.50\ kN}$

2.4.1

Since the line passing through points 7 and 8 represents an equal pressure surface;

$P_7 = P_8$ or $(h_{water})(\gamma_{water}) = (h_{oil})(\gamma_{oil})$

However; $(h_{oil})(\gamma_{oil}) = (h_{oil})(\gamma_{water})(SG_{oil})$, thus

$\mathbf{h_{oil}} = (h_{water})/(SG_{oil}) = (52.3 \text{ cm})/(0.85) = \mathbf{61.5 \text{ cm}}$

2.4.2

A surface of equal pressure surface can be drawn at the mercury-water meniscus. Therefore,

$(3 \text{ ft})(\gamma_{water}) = (h)(\gamma_{Hg})$

$h = (3 \text{ ft})(\gamma_{water}/\gamma_{Hg}) = (3 \text{ ft}) / (SG_{Hg}) = (3 \text{ ft}) / (13.6)$

h = 0.221 ft = 2.65 in.

2.4.3

The pressure at the bottom registered by the gage is equal to the pressure due to the liquid heights. Thus,

$(h_{Hg})(SG_{Hg})(\gamma_{water}) = (4h)(\gamma_{water}) + (h)(SG_{oil})(\gamma_{water})$

$h = (h_{Hg})(SG_{Hg})/(4 + SG_{oil})$

$\mathbf{h} = (26.3 \text{ cm})(13.6)/(4 + 0.82) = \mathbf{74.2 \text{ cm}}$

2.4.4

A surface of equal pressure can be drawn at the mercury-water meniscus. Therefore,

$P_A + (y)(\gamma_{water}) = (h)(\gamma_{Hg})$

$P_A + (0.034 \text{ m})(\gamma_{water}) = (0.026 \text{ m})(\gamma_{Hg})$

$$P_A = (0.026 \text{ m})(13.6)(9{,}790 \text{ N/m}^3) - (0.034 \text{ m})(9{,}790 \text{ N/m}^3)$$

$\mathbf{P_A = 3{,}130 \text{ N/m}^2}$ **(Pascals)** $\mathbf{= 3.13 \text{ kN/m}^2}$

2.4.5

A surface of equal pressure can be drawn at the mercury-water meniscus. Therefore,

$P_{pipe} + (2 \text{ ft})(\gamma_{water}) = (h)(\gamma_{Hg})$

$(16.8 \text{ lb/in}^2)(144 \text{ in}^2/\text{ft}^2) + (2 \text{ ft})(\gamma_{water}) = (h)(\gamma_{Hg})$

$$(2.42 \times 10^3 \text{ lb/ft}^2) + (2 \text{ ft})(62.3 \text{ lb/ft}^3) = (h)(13.6)(62.3 \text{ lb/ft}^3)$$

h = 3.00 ft (manometer is correct)

2.4.6

Using the "swim through" technique, start at the end of the manometer which is open to the atmosphere and thus equal to zero gage pressure. Then "swim through" the manometer, adding pressure when "swimming" down and subtracting when "swimming" up until you reach the pipe. The computations are below:

$$0 - (0.66 \text{ m})(\gamma_{CT}) + [(0.66 + y + 0.58)\text{m}](\gamma_{air}) - (0.58 \text{ m})(\gamma_{oil}) = P_{pipe}$$

The specific weight of air is negligible when compared to fluids, so that term in the equation can be dropped.

$P_{pipe} = 0 - (0.66 \text{ m})(SG_{CT})(\gamma) - (0.58 \text{ m})(SG_{oil})(\gamma)$

$$P_{pipe} = 0 - (0.66 \text{ m})(1.60)(9790 \text{ N/m}^3) - (0.58 \text{ m})(0.82)(9790 \text{ N/m}^3)$$

$P_{pipe} = -15.0 \text{ kN/m}^2$ Pressure can be converted to height (head) of any liquid through $P = \gamma \cdot h$. Thus,

$\mathbf{h_{pipe}} = (-15{,}000 \text{ N/m}^2)/(9790 \text{ N/m}^3) = \mathbf{-1.53 \text{ m of water}}$

2.4.7

A surface of equal pressure surface can be drawn at the mercury-water meniscus. Therefore,

$P + (h_1)(\gamma) = (h_2)(\gamma_{Hg}) = (h_2)(SG_{Hg})(\gamma)$

$P + (0.575 \text{ ft})(62.3 \text{ lb/ft}^3) = (2.00 \text{ ft})(13.6)(62.3 \text{ lb/ft}^3)$

$\mathbf{P = 1{,}660 \text{ lb/ft}^2 = 11.5 \text{ psi}}$

2.4.8

A surface of equal pressure surface can be drawn at the mercury-water meniscus. Therefore,

$P_{pipe} + (h_1)(\gamma) = (h_2)(\gamma_{Hg}) = (h_2)(SG_{Hg})(\gamma)$

$P_{pipe} + (0.20\ m)(9790\ N/m^3) = (0.67\ m)(13.6)(9790\ N/m^3)$

$\mathbf{P_{pipe} = 8.72 \times 10^4\ N/m^2}$ **(Pascals) = 87.2 KPa**

When the manometer reading rises or falls, mass balance must be preserve in the system. Therefore,

$Vol_{res} = Vol_{tube}$ or $A_{res}\cdot h_1 = A_{tube}\cdot h_2$

$h_1 = h_2\ (A_{tube}/A_{res}) = h_2\ [(D_{tube})^2/(D_{res})^2]$

$\mathbf{h_1} = (10\ cm)[(0.5\ cm)^2/(5\ cm)^2] =$ **0.1 cm**

2.4.9

Using the "swim through" technique, start at pipe *A* and "swim through" the manometer, adding pressure when "swimming" down and subtracting when "swimming" up until you reach pipe B. The computations are:

$P_A + (5.33\ ft)(\gamma) - (1.67ft)(\gamma_{Hg}) - (1.0\ ft)(\gamma_{oil}) = P_B$

$P_A - P_B = (62.3\ lb/ft^3)\ [(1.0\ ft)(0.82) - (5.33\ ft) + (1.67\ ft)(13.6)]$

$\mathbf{P_A - P_B = 1{,}130\ lb/ft^2 = 7.85\ psi}$

2.4.10

Using the "swim through" technique, start at P_2 and "swim through" the manometer, adding pressure when "swimming" down and subtracting when "swimming" up until you reach P_1. The computations are:

$P_2 + (\Delta h)(\rho_1\cdot g) + (y)(\rho_1\cdot g) + (h)(\rho_2\cdot g) - (h)(\rho_1\cdot g) - (y)(\rho_1\cdot g) = P_1$

where y is the vertical elevation difference between the fluid surface in the left hand reservoir and the interface between the two fluids on the right side of the U-tube.

$P_1 - P_2 = (\Delta h)(\rho_1\cdot g) + (h)(\rho_2\cdot g) - (h)(\rho_1\cdot g)$

2.4.10 – cont.

When the manometer reading (h) rises or falls, mass balance must be preserve in the system. Therefore,

$Vol_{res} = Vol_{tube}$ or $A_{res}\cdot(\Delta h) = A_{tube}\cdot h$

$\Delta h = h\ (A_{tube}/A_{res}) = h\ [(d_2)^2/(d_1)^2]$; substituting yields

$\mathbf{P_1 - P_2 = h\ [(d_2)^2/(d_1)^2]\ (\rho_1\cdot g) + (h)(\rho_2\cdot g) - (h)(\rho_1\cdot g)}$

$\mathbf{P_1 - P_2 = h\cdot g\ [\rho_2 - \rho_1 + \rho_1\ \{(d_2)^2/(d_1)^2\}]}$

$\mathbf{P_1 - P_2 = h\cdot g\ [\rho_2 - \rho_1\ \{1 - (d_2)^2/(d_1)^2\}]}$

2.4.11

Using the "swim through" technique, start at both ends of the manometers which are open to the atmosphere and thus equal to zero gage pressure. Then "swim through" the manometer, adding pressure when "swimming" down and subtracting when "swimming" up until you reach the pipes in order to determine P_A and P_B. The computations are below:

$0 + (23)(13.6)(\gamma) - (44)(\gamma) = P_A$; $P_A = 269\cdot\gamma$

$0 + (46)(0.8)(\gamma) + (20)(13.6)(\gamma) - (40)(\gamma) = P_B$

$P_B = 269\cdot\gamma$; Therefore, $P_A = P_B$ and $\mathbf{h = 0}$

2.4.12

Using the "swim through" technique, start at the sealed right tank where the pressure is known. Then "swim through" the tanks and pipes, adding pressure when "swimming" down and subtracting when "swimming" up until you reach the left tank where the pressure is not known. The computations are as follows:

$20\ kN/m^2 + (4.5\ m)(9.79\ kN/m^3) - (2.5\ m)(1.6)(9.79\ kN/m^3) - (5\ m)(0.8)(9.79\ kN/m^3) = P_{left}$

$\mathbf{P_{left} = -14.3\ kN/m^2}$ **(or -14.3 kPa)**

$P_B = (-14.3\ kN/m^2) / [(SG_{Hg})(\gamma)]$

$P_B = (-14.3\ kN/m^2) / [(13.6)(9.79\ kN/m^3)]$

$\mathbf{P_B = -0.107\ m = 10.7\ cm\ (Hg)}$

2.5.1

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)[(3m)/(3)]·[6 m^2]

F = 5.87 x 10^4 N = 58.7 kN

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(4m)(3m)^3/36]}{[(4m)(3m)/2](1.00m)} + 1.00m$$

y_p = 1.50 m (depth to center of pressure)

2.5.2

$F = \gamma \cdot \bar{h} \cdot A$ = (62.3 lb/ft^3)[(30 ft)/2]·[(30 ft)(1 ft)]

F = 2.80 x 10^4 lbs per foot of length

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(1ft)(30ft)^3/12]}{[(30ft)(1ft)](15ft)} + 15ft$$

y_p = 20.0 ft (depth to the center of pressure)

In summing moments about the toe of the dam ($\sum M_A$), the weight acts to stabilize the dam (called a righting moment) and the hydrostatic force tends to tip it over (overturning moment).

M = (Wt.)[(2/3)(10)] - F(10ft) =

[1/2 (10 ft)(30 ft)·(1 ft)](2.67)(62.3 lb/ft^3)·(6.67 ft) –

(2.80 x 10^4 lbs)·(10 ft) = -1.14 x 10^5 ft-lbs

M = 1.14 x 10^5 ft-lbs (overturning; dam is unsafe)

2.5.3

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(1 m)(π)(0.5 m)^2

F = 7.69 x 10^3 N = 7.69 kN ; $\bar{y} = \bar{h}/\sin 45°$;

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[\pi(1m)^4/64]}{[\pi(1m)^2/4](1.414m)} + 1.414m$$

y_p = 1.46 m (distance from water surface to the center of pressure along the incline).

2.5.4

$F_{square} = \gamma \cdot \bar{h} \cdot A$ = γ(L/2)(L^2) = (γ/2)·L^3

$F_{tri} = \gamma \cdot \bar{h} \cdot A$ = γ(L+H/3)(LH/2) = (γ/2)[L^{2}H + LH^2/3]

Setting the two forces equal: $F_{square} = Y_{tri}$;

(γ/2)·L^3 = (γ/2)[L^{2}H + LH^2/3]

L^2 - HL - H^2/3 = 0; divide by H^2 and solve quadratic

(L/H)^2 – (L/H) - 1/3 = 0; L/H = 1.26 or **H/L = 0.791**

2.5.5

$F_{left} = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(0.5 m)[(1.41m)(3m)]

F_{left} = 20.7 kN (where A is "wet" surface area)

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(3m)(1.41m)^3/12]}{[(3m)(1.41m)](0.705m)} + 0.705m$$

y_p = 0.940 m (inclined distance to center of pressure)

Location of this force from the hinge (moment arm):

Y' = 2 m – 1.41 m + 0.940 m = 1.53 m

$F_{right} = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(h/2 m)[(h/cos45°)(3m)]

F_{right} = 20.8·h^2 kN

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(3)(1.41\cdot h)^3/12]}{[(3)(1.41\cdot h)](0.705\cdot h)} + 0.705\cdot h$$

y_p = (0.940·h)m; Moment arm of force from hinge:

Y" = 2 m – (h/sin 45)m + (0.940·h)m = 2m – (0.474·h)m

The force due to the gate weight: W = 20.0 kN
Moment arm of this force from hinge: X = 0.707 m

Summing moments about the hinge yields: $\sum M_{hinge}$= 0

(20.8·h^2)[2m – (0.474·h)] – 20.7(1.53) – 20(0.707) = 0

h = 1.25 m (gate opens when depth exceeds 1.25 m)

2.5.6

$F = \gamma \cdot \bar{h} \cdot A$ = (62.3 lb/ft^3)[(7 ft)]·[π(6 ft)^2/4]

F = 1.23 x 10^4 lbs

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[\pi(6ft)^4/64]}{[\pi(6ft)^2/4](7ft)} + 7ft$$

y_p = 7.32 ft (depth to the center of pressure)

Thus, summing moments: $\sum M_{hinge} = 0$

P(3 ft) – (1.23 x 10^4 lbs)(0.32 ft) = 0

P = 1.31 x 10^3 lbs

2.5.7

In order for the balance to be maintained at h = 4 feet, the center of pressure should be at the pivot point (i.e., the force at the bottom check block is zero). As the water rises above h = 4 feet, the center of pressure will rise above the pivot point and open the gate. Below h = 4 feet, the center of pressure will be lower than the pivot point and the gate will remain closed. Thus, for a unit width of gate, the center of pressure is

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(1ft)(10ft)^3/12]}{[(1ft)(10ft)](9ft)} + 9ft$$

y_p = 9.93 ft (vertical distance from water surface to the center of pressure)

Thus, the horizontal axis of rotation (0-0') should be 14 ft – 9.93 ft = 4.07 ft above the bottom of the gate.

2.5.8

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(2.5 m)[(π){(1.5)^2–(0.5)^2}m^2]

F = 1.54 x 10^5 N = 154 kN

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[\pi(3m)^4/64 - \pi(1m)^4/64)]}{[\pi(3m)^2/4 - \pi(1m)^2/4](2.5m)} + 2.5m$$

y_p = 2.75 m (below the water surface)

2.5.9

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(2.5 m)[(π)(1.5 m)^2–(1.0 m)^2]

F = 1.49 x 10^5 N = 149 kN

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[\pi(3m)^4/64 - (1m)(1m)^3/12)]}{[\pi(1.5m)^2 - (1m)^2](2.5m)} + 2.5m$$

y_p = 2.76 m (below the water surface)

2.5.10

$F = \gamma \cdot \bar{h} \cdot A$ = (9790 N/m^3)(2.5m)[(5/cos 30°)(3 m)]

F = 4.24 x 10^5 N = 424 kN

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(3m)(5.77m)^3/12]}{[(3m)(5.77m)](2.89m)} + 2.89m$$

y_p = 3.85 m (inclined depth to center of pressure)

Summing moments about the base of the dam; $\sum M = 0$

(424 kN)(5.77 m – 3.85 m) – (F_{AB})(5.77m/2) = 0

F_{AB} = 282 kN

2.5.11

$F = \gamma \cdot \bar{h} \cdot A$ = (62.3 lb/ft^3)[(*d*/2)ft]·[{(*d* /cos30)ft}(8 ft)]

F = 288·d^2 lbs

$$y_P = \frac{I_0}{A\bar{y}} + \bar{y} = \frac{[(8)(d/\cos 30°)^3/12]}{[(8)(d/\cos 30°)](d/2\cos 30°)} + (d/2\cos 30)$$

y_p = [(0.192·*d*) + 0.577·*d*)] ft = 0.769·*d* (inclined depth)

Thus, summing moments: $\sum M_{hinge} = 0$

(288·d^2)[(*d*/cos30)-0.769*d*] – (5,000)(15) = 0

d = 8.77 ft A depth greater than this will make the gate open, and anything less will make it close.