

# Chapter 11

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11.1

$$(a) m = \frac{6.8 \text{ N}}{1.62 \text{ m/s}^2} = 4.198 \text{ kg} \quad \blacktriangleleft$$

$$(b) W = mg = 4.198(9.81) = 41.2 \text{ kg} \quad \blacktriangleleft$$

11.2

$$W = \frac{1}{3}\pi R^2 h \rho g = \frac{\pi}{3}(0.075^2)(0.125)(2700)(9.81) = 19.503 \text{ N} \quad \blacktriangleleft$$

11.3

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}mk^2\omega^2$$

Since the dimensions of each term must be the same, we have

$$[KE] = [M] \left[ \frac{L^2}{T^2} \right] = [M][k^2] \left[ \frac{1}{T^2} \right]$$

Therefore,

$$[k] = [L]$$

(a) In the SI system

$$[KE] = [M] \left[ \frac{L^2}{T^2} \right] = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \quad \blacktriangleleft \quad [k] = \text{m} \quad \blacktriangleleft$$

11.4

$$[g][k][x] \left[ \frac{1}{W} \right] = \left[ \frac{L}{T^2} \right] \left[ \frac{F}{L} \right] [L] \left[ \frac{1}{F} \right] = \left[ \frac{L}{T^2} \right] = [a] \text{ Q.E.D}$$

11.5

(a)

$$[\delta] = \left[ \frac{PL}{EA} \right] \quad [L] = \left[ \frac{1}{E} \right] \left[ \frac{FL}{L^2} \right] \quad [E] = \left[ \frac{F}{L^2} \right] \quad \blacktriangleleft$$

(b) Substituting  $[F] = [ML/T^2]$  into the result of part (a):

$$[E] = \left[ \frac{ML}{T^2} \right] \left[ \frac{1}{L^2} \right] = \left[ \frac{M}{T^2L} \right] \blacktriangleleft$$

**11.6** (a)  $[mv^2] = \left[ \frac{FT^2}{L} \right] \left[ \frac{L^2}{T^2} \right] = [FL] \blacktriangleleft$

(b)  $[mv] = \left[ \frac{FT^2}{L} \right] \left[ \frac{L}{T} \right] = [FT] \blacktriangleleft$

(c)  $[ma] = \left[ \frac{FT^2}{L} \right] \left[ \frac{L}{T^2} \right] = [F] \blacktriangleleft$

**11.7** Rewrite the equation as  $y = 1.0 x^2$

$$[y] = [1.0][x^2] \quad [L] = [1.0][L^2] \quad [1.0] = \left[ \frac{1}{L} \right]$$

$y = x^2$  can be dimensionally correct only if the units of the implied constant 1.0 are  $\text{cm}^{-1}$ .  $\blacktriangleleft$

**11.8** (a)  $[I] = [mR^2] = \left[ \frac{FT^2}{L} \right] [L^2] = [FLT^2] \blacktriangleleft$

(b)  $[I] = [mR^2] = [ML^2] \blacktriangleleft$

**11.9** (a)  $[v^3] = [A][x^2] + [B][v][t^2] \quad \left[ \frac{L^3}{T^3} \right] = [A][L^2] + [B] \left[ \frac{L}{T} \right] [T^2]$

$$[A] = \left[ \frac{L}{T^3} \right] \blacktriangleleft \quad [B] = \left[ \frac{L^2}{T^4} \right] \blacktriangleleft$$

(b)  $[x^2] = [A][t^2] \left[ e^{[B][t^2]} \right] \quad [L^2] = [A][T^2][1] \quad [B][T^2] = [1]$

$$[A] = \left[ \frac{L^2}{T^2} \right] \blacktriangleleft \quad [B] = \left[ \frac{1}{T^2} \right] \blacktriangleleft$$

**11.10**

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = P_0 \sin \omega t$$

$$[m] \left[ \frac{d^2x}{dt^2} \right] = \left[ \frac{FT^2}{L} \right] \left[ \frac{L}{T^2} \right] = [F]$$

Therefore, the dimension of each term in the expression is  $[F]$ .

$$\begin{aligned}
 [c] \left[ \frac{dx}{dt} \right] &= [c] \left[ \frac{L}{T} \right] = [F] & [c] &= \left[ \frac{FT}{L} \right] \blacktriangleleft \\
 [k][x] &= [k][L] = [F] & [k] &= \left[ \frac{F}{L} \right] \blacktriangleleft \\
 [P_0][\sin \omega t] &= [P_0][1] = [F] & [P_0] &= [F] \blacktriangleleft \\
 [\omega][t] &= [\omega][T] = [1] & [\omega] &= \left[ \frac{1}{T} \right] \blacktriangleleft
 \end{aligned}$$

### 11.11

$$F = G \frac{m_A m_B}{R^2} \quad G = \frac{FR^2}{m_A m_b} \quad [G] = \frac{[F][L^2]}{[M^2]}$$

$$(a) [G] = \frac{[F][L^2]}{[FT^2/L]^2} = \left[ \frac{L^4}{FT^4} \right] \blacktriangleleft$$

$$(b) [G] = \frac{[ML/T^2][L^2]}{[M^2]} = \left[ \frac{L^3}{MT^2} \right] \blacktriangleleft$$

**11.12** Using the base dimensions of an absolute  $[MLT]$  system:

$$[F] = [C][\rho][v^2][A] \quad \left[ \frac{ML}{T^2} \right] = [C] \left[ \frac{M}{L^3} \right] \left[ \frac{L^2}{T^2} \right] [L^2] \quad [C] = [1] \text{ Q.E.D } \blacktriangleleft$$

### 11.13

$$\begin{aligned}
 F &= G \frac{m^2}{R^2} = (6.67 \times 10^{-11}) \frac{8^2}{0.4^2} = 2.668 \times 10^{-8} \text{ N} \\
 W &= mg = 8(9.81) = 78.48 \text{ N} \\
 \frac{F}{W} \times 100\% &= \frac{2.668 \times 10^{-8}}{78.48} \times 100\% = 3.40 \times 10^{-8}\% \blacktriangleleft
 \end{aligned}$$

### 11.14

$$F = G \frac{m^2}{R^2} = (6.67 \times 10^{-11}) \frac{(0.907)^2}{(0.406)^2} = 3.33 \times 10^{-10} \text{ N } \blacktriangleleft$$

### 11.15

$$m = \frac{WR^2}{GM_e} = \frac{(3000)(6378 + 1600)^2 \times 10^6}{(6.67 \times 10^{-11})(5.9742 \times 10^{24})} = 479 \text{ kg } \blacktriangleleft$$

11.16

$$g_m = \frac{GM_m}{R_m^2} \quad g_e = \frac{GM_e}{R_e^2}$$
$$\frac{g_m}{g_e} = \frac{M_m}{M_e} \left( \frac{R_e}{R_m} \right)^2 = \frac{0.073483}{5.9742} \left( \frac{6378}{1738} \right)^2 = 0.1656 \approx \frac{1}{6} \text{ Q.E.D}$$

11.17

$$M_e = 5.9742 \times 10^{24} \text{ kg}$$

$$R_e = 6378 \times 10^3 \text{ m}$$

$$W = G \frac{M_e m}{(2R_e)^2} = 3.44 \times 10^{-8} \frac{(5.9742 \times 10^{24})(667/9.81)}{(2 \times 6378 \times 10^3)^2} = 166.6 \text{ N} \blacktriangleleft$$

11.18

$$F = G \frac{M_s m}{R^2} = 6.67 \times 10^{-11} \frac{(1.9891 \times 10^{30})(1.0)}{(149.6 \times 10^9)^2} = 0.00593 \text{ N} \blacktriangleleft$$

11.19

$$G \frac{M_e m}{r^2} = G \frac{M_s m}{(R-r)^2} \quad \frac{M_e}{r^2} = \frac{M_s}{(R-r)^2}$$
$$\frac{M_e}{M_s} = \frac{r^2}{R^2 - 2Rr + r^2}$$
$$\frac{5.9742 \times 10^{24}}{1.9891 \times 10^{30}} = \frac{r^2}{(149.6 \times 10^9)^2 - 2(149.6 \times 10^9)r + r^2}$$
$$0 = 2.238 \times 10^{22} - 2.992 \times 10^{11}r - 3.3294 \times 10^5 r^2$$
$$r = 259 \times 10^6 \text{ m} = 259 \times 10^3 \text{ km} \blacktriangleleft$$

# Chapter 12

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## 12.1

$$\begin{aligned}y &= -0.16t^4 + 4.9t^3 + 0.14t^2 \text{ m} \\v &= \dot{y} = -0.64t^3 + 14.7t^2 + 0.28t \text{ m/s} \\a &= \dot{v} = -1.92t^2 + 29.4t + 0.28 \text{ m/s}^2\end{aligned}$$

At maximum velocity ( $a = 0$ ):

$$-1.92t^2 + 29.4t + 0.28 = 0 \quad t = 15.322 \text{ s}$$

$$\begin{aligned}v_{\max} &= -0.64(15.322^3) + 14.7(15.322^2) + 0.28(15.322) \\&= 1153 \text{ m/s} \quad \blacktriangleleft \\y &= -0.16(15.322^4) + 4.9(15.322^3) + 0.14(15.322^2) \\&= 8840 \text{ m} \quad \blacktriangleleft\end{aligned}$$

## 12.2

$$(a) \quad x = -\frac{1}{2}gt^2 + v_0t \quad \therefore v = \dot{x} = -gt + v_0 \quad \blacklozenge \quad \therefore a = \ddot{x} = -g \quad \blacklozenge$$

When  $t = 0$ , then  $x = 0$  and  $v = v_0$ . Hence  $v_0$  is the initial velocity.

Since gravity is the only source of acceleration in this problem,  $g$  must be the gravitation acceleration.

$$(b) \quad \text{When } x = x_{\max} \text{ then } v = 0. \quad \therefore -gt + v_0 = 0 \quad \therefore t = \frac{v_0}{g}$$

$$\therefore x_{\max} = -\frac{1}{2}g \left(\frac{v_0}{g}\right)^2 + v_0 \left(\frac{v_0}{g}\right) = \frac{v_0^2}{2g} \quad \blacklozenge$$

$$\text{At the end of flight } x = 0. \quad \therefore -\frac{1}{2}gt^2 + v_0t = 0 \quad \therefore t = \frac{2v_0}{g} \quad \blacklozenge$$

$$(c) \quad \therefore x_{\max} = \frac{(26.8)^2}{2(9.81)} = 36.6 \text{ m} \quad \blacklozenge \quad \therefore t = \frac{2(26.8)}{9.81} = 5.46 \text{ s} \quad \blacklozenge$$

## 12.3

$$\begin{aligned}x &= 6(1 - e^{-t/2}) \text{ m} \\v &= \dot{x} = 6 \left(\frac{1}{2}e^{-t/2}\right) = 3e^{-t/2} \text{ m/s} \\a &= \dot{v} = -3 \left(\frac{1}{2}e^{-t/2}\right) = -1.5e^{-t/2} \text{ m/s}^2\end{aligned}$$

(a)  $x_{\max} = 6 \text{ m}$  at  $t = \infty$  ◀  
 $v_{\max} = 3 \text{ m/s}$  and  $|a|_{\max} = 1.5 \text{ m/s}^2$  both occurring at  $t = 0$  ◀

(b) When  $x = 3 \text{ m}$ :  $3 = 6(1 - e^{-t/2})$      $e^{-t/2} = 0.5$   
 $t = -2 \ln(0.5) = 1.3863 \text{ s}$  ◀  
 $v = 3(0.5) = 1.5 \text{ m/s}$  ◀     $a = -1.5(0.5) = -0.75 \text{ m/s}^2$  ◀

## 12.4

$$x = t^3 - 6t^2 - 32t \text{ m}$$

$$v = \dot{x} = 3t^2 - 12t - 32 \text{ m/s}$$

$$a = \dot{v} = 6t - 12 \text{ m/s}^2$$

At  $t = 10 \text{ s}$ :

$$x = 10^3 - 6(10^2) - 32(10) = 80 \text{ m} \quad \blacktriangleleft$$

$$v = 3(10^2) - 12(10) - 32 = 148 \text{ m/s} \quad \blacktriangleleft$$

$$a = 6(10) - 12 = 48 \text{ m/s}^2 \quad \blacktriangleleft$$

Reversal of velocity occurs when  $v = 0$  ( $t \neq 0$ ):

$$v = 3t^2 - 12t - 32 = 0 \quad t = 5.830 \text{ s}$$

$$x = 5.830^3 - 6(5.830^2) - 32(5.830) = -192.3 \text{ m}$$

At  $t = 10 \text{ s}$  the distance travelled is

$$s = 192.3 + (192.3 + 80) = 466 \text{ m} \quad \blacktriangleleft$$

12.5 (a)  $x = t^2 - \frac{t^3}{90} \text{ m}$      $v = \dot{x} = 2t - \frac{t^2}{30} \text{ m/s}$      $v = 0$  when  $t = 60 \text{ s}$

$$x_{\max} = 60^2 - \frac{60^3}{90} = 1200 \text{ m} \quad \blacktriangleleft$$

(b)  $a = \dot{v} = 2 - \frac{t}{15} \text{ m/s}^2$      $a = 0$  when  $t = 30 \text{ s}$

$$v_{\max} = 2(30) - \frac{30^2}{30} = 30 \text{ m/s} \quad \blacktriangleleft$$

12.6

$$(a) x = v_0(t - t_0 + t_0 e^{-t/t_0}) \quad \therefore v = \dot{x} = v_0(1 - e^{-t/t_0}) \blacklozenge$$

Since  $v \rightarrow v_0$  as  $t \rightarrow \infty$ ,  $v_0$  is the limiting or terminal velocity.

$$(b) a = \dot{v} = \frac{v_0}{t_0} e^{-t/t_0} \blacklozenge \quad \text{But from part (a): } v_0 - v = v_0 e^{-t/t_0} \quad \therefore a = \frac{v_0 - v}{t_0} \blacklozenge$$

12.7

$$x = 3t^2 - 12t \text{ m} \quad v = \dot{x} = 6t - 12 \text{ m/s}$$

(a) The bead leaves the wire when  $x = 40$  m

$$3t^2 - 12t = 40 \quad t = 6.16 \text{ s} \blacktriangleleft$$

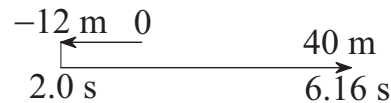
(b) Reversal of velocity occurs when  $v = 0$  ( $t \neq 0$ ):

$$v = 6t - 10 \quad t = 2.0 \text{ s}$$

$$x = 3(2.0^2) - 12(2.0) = -12.0 \text{ m}$$

The distance travelled is

$$s = 2(12) + 40 = 64.0 \text{ m} \blacktriangleleft$$



12.8

$$x = 4t^2 - 2 \text{ mm}$$

$$y = \frac{x^2}{12} = \frac{16t^4 - 16t^2 + 4}{12} = \frac{4t^4 - 4t^2 + 1}{3} \text{ mm}$$

When  $t = 2$  s:

$$v_x = \dot{x} = 8t = 8(2) = 16 \text{ mm/s}$$

$$v_y = \dot{y} = \frac{16t^3 - 8t}{3} = \frac{16(2)^3 - 8(2)}{3} = 37.33 \text{ mm/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{16^2 + 37.33^2} = 40.6 \text{ mm/s} \blacktriangleleft$$

$$a_x = \dot{v}_x = 8 \text{ mm/s}^2$$

$$a_y = \dot{v}_y = \frac{48t^2 - 8}{3} = \frac{48(2)^2 - 8}{3} = 61.33 \text{ mm/s}^2$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8^2 + 61.33^2} = 61.9 \text{ mm/s}^2 \blacktriangleleft$$

## 12.9

$$(a) \mathbf{x} = R \left( 1 + \frac{1}{2} \cos \omega t \right) \quad \therefore \mathbf{v} = \dot{\mathbf{x}} = -\frac{1}{2} R \omega \sin \omega t \quad \therefore \mathbf{a} = \dot{\mathbf{v}} = -\frac{1}{2} R \omega^2 \cos \omega t \quad \blacklozenge$$

$$(b) |v|_{\max} = \frac{1}{2} R \omega \quad |a|_{\max} = \frac{1}{2} R \omega^2$$

$\therefore$  Doubling  $\omega$  would double  $|v|_{\max}$  and quadruple  $|a|_{\max}$ .

## 12.10

$$y = 50 - 2t \text{ s} \quad v_y = \dot{y} = -2 \text{ m/s} \quad a_y = \dot{v}_y = 0$$

$$x = \frac{6}{y} = \frac{6}{50 - 2t} \text{ m} \quad v_x = \dot{x} = \frac{3}{(25 - t)^2} \text{ m/s} \quad a_x = \dot{v}_x = \frac{6}{(25 - t)^3}$$

At  $t = 20$  s:

$$v_x = \frac{3}{(25 - 20)^2} = 0.12 \text{ m/s} \quad v_y = -2 \text{ m/s} \quad \mathbf{v} = 0.12\mathbf{i} - 2\mathbf{j} \text{ m/s} \quad \blacktriangleleft$$

$$a_x = \frac{6}{(25 - 20)^3} = 0.048 \text{ m/s}^2 \quad a_y = 0 \quad \mathbf{a} = 0.048\mathbf{i} \text{ m/s}^2 \quad \blacktriangleleft$$

## 12.11

$$(a) v^2 = 2gr_0(r_0/r - 1) + v_0^2$$

Differentiate with respect to time:  $2v\dot{v} = 2gr_0(-r_0/r^2)\dot{r}$  or  $2va = -2g(r_0/r)^2\dot{r}$

$$\therefore \mathbf{a} = -g(r_0/r)^2 \dot{r} \quad \blacklozenge$$

(b)  $v_0$  is the escape velocity if  $v \rightarrow 0$  when  $r \rightarrow \infty$ .

$$\therefore 0 = \lim_{r \rightarrow \infty} [2gr_0(r_0/r - 1) + v_0^2] \quad \therefore 0 = 2gr_0(0 - 1) + v_0^2 \quad \therefore v_0 = \sqrt{2gr_0} \quad \blacklozenge$$

$$(c) \text{ For earth: } v_0 = \sqrt{2(9.81)(6340 \times 1000)} = 11153 \text{ m/s} \quad \blacklozenge$$

## 12.12

$$x = 15 - 2t^2 \text{ m} \quad v_x = \dot{x} = -4t \text{ m/s} \quad a_x = \dot{v}_x = -4 \text{ m/s}^2$$

$$y = 15 - 10t + t^2 \text{ m} \quad v_y = \dot{y} = -10 + 2t \text{ m/s} \quad a_y = \dot{v}_y = 2 \text{ m/s}^2$$

$$(a) \text{ At } t = 0: \quad \mathbf{v} = -10\mathbf{j} \text{ m/s} \quad \blacktriangleleft \quad \mathbf{a} = -4\mathbf{i} + 2\mathbf{j} \text{ m/s}^2 \quad \blacktriangleleft$$

$$(b) \text{ At } t = 5 \text{ s:} \quad \mathbf{v} = -20\mathbf{i} \text{ m/s} \quad \blacktriangleleft \quad \mathbf{a} = -4\mathbf{i} + 2\mathbf{j} \text{ m/s}^2 \quad \blacktriangleleft$$



### 12.13

$$x = 58t \text{ m} \quad v_x = \dot{x} = 58 \text{ m/s} \quad a_x = \dot{v}_x = 0$$

$$y = 78t - 4.91t^2 \text{ m} \quad v_y = \dot{y} = 78 - 9.82t \text{ m/s} \quad a_y = \dot{v}_y = -9.82 \text{ m/s}^2$$

(a)  $\mathbf{a} = -9.82\mathbf{j} \text{ m/s}^2 \blacktriangleleft$

(b)  $\mathbf{v}|_{t=0} = 58\mathbf{i} + 78\mathbf{j} \text{ m/s} \blacktriangleleft$

(c)  $y = h$  when  $v_y = 0$ :

$$v_y = 78 - 9.82t = 0 \quad t = 7.943 \text{ s}$$

$$h = 78(7.943) - 4.91(7.943^2) = 310 \text{ m} \blacktriangleleft$$

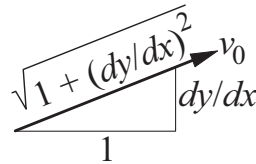
(d)  $x = L$  when  $y = -140 \text{ m}$ :

$$y = 78t - 4.91t^2 = -140 \quad t = 17.514 \text{ s}$$

$$L = 58(17.514) = 1016 \text{ m} \blacktriangleleft$$

### 12.14

$$y = \frac{x^2}{1000} \quad \frac{dy}{dx} = \frac{x}{500}$$



$$v_x = v_0 \frac{1}{\sqrt{1 + (dy/dx)^2}} = \frac{v_0}{\sqrt{1 + (x/500)^2}} = \frac{500v_0}{\sqrt{500^2 + x^2}}$$

$$v_y = v_0 \frac{dy/dx}{\sqrt{1 + (dy/dx)^2}} = \frac{x/500}{\sqrt{1 + (x/500)^2}} = \frac{xv_0}{\sqrt{500^2 + x^2}}$$

When  $x = 100 \text{ m}$ :

$$v_x = \frac{500(6)}{\sqrt{500^2 + 100^2}} = 5.883 \text{ m/s}$$

$$v_y = \frac{100(6)}{\sqrt{500^2 + 100^2}} = 1.1767 \text{ m/s}$$

$$\mathbf{v} = 5.88\mathbf{i} + 1.177\mathbf{j} \text{ m/s} \blacktriangleleft$$

$$a_x = \dot{v}_x = \frac{dv_x}{dx} \frac{dx}{dt} = \frac{dv_x}{dx} v_x = -\frac{500xv_0}{(500^2 + x^2)^{3/2}} v_x$$

$$a_y = \dot{v}_y = \frac{dv_y}{dx} \frac{dx}{dt} = \frac{dv_y}{dx} v_x = \frac{500^2 v_0}{(500^2 + x^2)^{3/2}} v_x$$

When  $x = 100$  m:

$$a_x = -\frac{500(100)(6)}{(500^2 + 100^2)^{3/2}} (5.883) = -0.01331 \text{ m/s}^2$$

$$a_y = \frac{500^2(6)}{(500^2 + 100^2)^{3/2}} (5.883) = 0.0666 \text{ m/s}^2$$

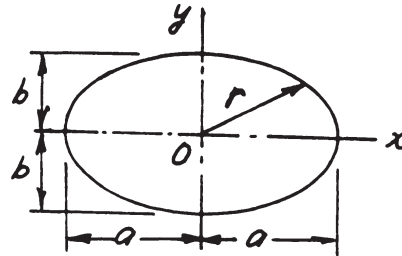
$$\mathbf{a} = -0.01332\mathbf{i} + 0.0666\mathbf{j} \text{ m/s}^2 \quad \blacktriangleleft$$

12.15

(a)  $x = a \cos \omega t \quad y = b \sin \omega t$

$$\therefore \frac{x^2}{a^2} + \frac{y^2}{b^2} = \cos^2 \omega t + \sin^2 \omega t = 1$$

which is the equation of ellipse  $\therefore$  Q.E.D.



(b)  $\mathbf{v}_x = \dot{x} = -a\omega \sin \omega t \quad \mathbf{a}_x = \ddot{x} = -a\omega^2 \cos \omega t = -\omega^2 x$

$$\mathbf{v}_y = \dot{y} = b\omega \cos \omega t \quad \mathbf{a}_y = \ddot{y} = -b\omega^2 \sin \omega t = -\omega^2 y$$

$$\therefore \mathbf{a} = a_x \mathbf{i} + a_y \mathbf{j} = -\omega^2(x\mathbf{i} + y\mathbf{j}) = -\omega^2 \mathbf{r} \quad \therefore \mathbf{a} \text{ and } \mathbf{r} \text{ are collinear} \quad \therefore \text{Q.E.D.}$$

12.16

$$x = R \cos \omega t + R\omega t \sin \omega t \quad y = R \sin \omega t - R\omega t \cos \omega t$$

$$\therefore \mathbf{v}_x = \dot{x} = -R\omega \sin \omega t + R\omega \sin \omega t + R\omega^2 t \cos \omega t = R\omega^2 t \cos \omega t$$

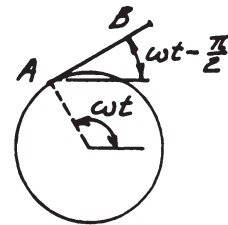
$$\therefore \mathbf{v}_y = \dot{y} = R\omega \cos \omega t - R\omega \cos \omega t + R\omega^2 t \sin \omega t = R\omega^2 t \sin \omega t$$

$$\therefore \mathbf{v} = v_x \mathbf{i} + v_y \mathbf{j} = R\omega^2 t (\mathbf{i} \cos \omega t + \mathbf{j} \sin \omega t) \quad \therefore \mathbf{v} = R\omega^2 t \quad \blacklozenge$$

$$\overrightarrow{AB} = \overline{AB} [\mathbf{i} \cos(\omega t - \pi/2) + \mathbf{j} \sin(\omega t - \pi/2)]$$

$$= \overline{AB} (\mathbf{i} \sin \omega t - \mathbf{j} \cos \omega t)$$

By inspection:  $\mathbf{v} \cdot \overrightarrow{AB} = 0 \quad \therefore \mathbf{v}$  is perpendicular to  $AB \quad \therefore$  Q.E.D.



12.17

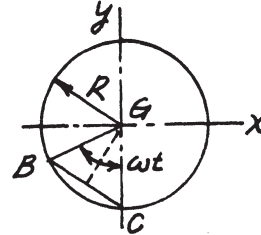
$$(a) \mathbf{x} = R(\omega t - \sin \omega t) \quad \therefore \mathbf{v}_x = \dot{\mathbf{x}} = R\omega(1 - \cos \omega t)$$

$$y = R(1 - \cos \omega t) \quad \therefore \mathbf{v}_y = \dot{y} = R\omega \sin \omega t$$

$$\therefore v^2 = v_x^2 + v_y^2 = R^2\omega^2[(1 - \cos \omega t)^2 + \sin^2 \omega t] = 2R^2\omega^2(1 - \cos \omega t) = 4R^2\omega^2 \sin^2 \frac{\omega t}{2}$$

$$\therefore v = 2R\omega \sin \frac{\omega t}{2}$$

$$\text{But } \overline{BC} = 2R \sin \frac{\omega t}{2} \quad \therefore v = \omega \overline{BC} \quad \therefore \text{Q.E.D.}$$



$$(b) \mathbf{a}_x = \ddot{\mathbf{x}} = R\omega^2 \sin \omega t \quad \mathbf{a}_y = \ddot{y} = R\omega^2 \cos \omega t$$

$$\therefore \mathbf{a} = a_x \mathbf{i} + a_y \mathbf{j} = (R\omega^2 \sin \omega t)\mathbf{i} + (R\omega^2 \cos \omega t)\mathbf{j}$$

$$\overrightarrow{BG} = (R \sin \omega t)\mathbf{i} + (R \cos \omega t)\mathbf{j} = \frac{1}{\omega^2} \mathbf{a} \quad \therefore \overrightarrow{BG} \text{ and } \mathbf{a} \text{ are parallel} \quad \therefore \text{Q.E.D.}$$

12.18

$$x = R \cos \omega t \quad y = R \sin \omega t \quad z = -\frac{h}{2\pi} \omega t$$

$$\therefore \mathbf{v}_x = \dot{x} = -R\omega \sin \omega t \quad \mathbf{v}_y = \dot{y} = R\omega \cos \omega t \quad \mathbf{v}_z = \dot{z} = -\frac{h\omega}{2\pi}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2 = R^2\omega^2 \sin^2 \omega t + R^2\omega^2 \cos^2 \omega t + \left(\frac{h\omega}{2\pi}\right)^2 = (R\omega)^2 + \left(\frac{h\omega}{2\pi}\right)^2$$

$$\therefore v = R\omega \sqrt{1 + \left[\frac{h}{2\pi R}\right]^2} = \text{constant} \quad \therefore \text{Q.E.D.}$$

$$\mathbf{a}_x = \ddot{x} = -R\omega^2 \cos \omega t \quad \mathbf{a}_y = \ddot{y} = -R\omega^2 \sin \omega t \quad \mathbf{a}_z = \ddot{z} = 0$$

$$a^2 = a_x^2 + a_y^2 + a_z^2 = R^2\omega^4 (\cos^2 \omega t + \sin^2 \omega t) = R^2\omega^4 \quad \therefore a = R\omega^2 = \text{constant} \quad \therefore \text{Q.E.D.}$$

Using the given data:

$$v = (1.2)(4\pi) \sqrt{1 + \left[\frac{0.75}{2\pi(1.2)}\right]^2} = 15.15 \text{ m/s} \quad \mathbf{\blacklozenge} \quad a = (1.2)(4\pi)^2 = 189.5 \text{ m/s}^2 \quad \mathbf{\blacklozenge}$$

12.19

$$\begin{aligned}
 x &= 0.8v_0t & y &= 0.6v_0t & z &= -0.04v_0^2t^2 \\
 \therefore v_x = \dot{x} &= 0.8v_0 & v_y = \dot{y} &= 0.6v_0 & v_z = \dot{z} &= -0.08v_0^2t \\
 \therefore a_x = \ddot{x} &= 0 & a_y = \ddot{y} &= 0 & a_z = \ddot{z} &= -0.08v_0^2 \\
 \text{(a) At point B: } x &= 4 \text{ in} & \therefore 4 &= 0.8v_0t & \therefore t &= 5/v_0 & \therefore v_z &= -0.4v_0 \\
 \therefore v &= \sqrt{v_x^2 + v_y^2 + v_z^2} &= v_0\sqrt{0.8^2 + 0.6^2 + 0.4^2} &= 1.0770v_0 \quad \blacklozenge \\
 a &= |a_z| = 0.08v_0^2 \quad \blacklozenge
 \end{aligned}$$

(b) Let  $\theta$  be the angle between the path and the z-axis at point B.

$$\therefore \cos\theta = \frac{\mathbf{v} \cdot \mathbf{k}}{|\mathbf{v}|} = \frac{-0.4v_0}{1.0770v_0} = -0.3714 \quad \therefore \theta = 111.8^\circ$$

$\therefore$  The angle between the path and the x-y plane is:  $\theta - 90^\circ = 21.8^\circ \quad \blacklozenge$

12.20

$$\begin{aligned}
 \text{(a)} \quad \mathbf{r} &= (3t^2 + 4t)\mathbf{i} + (-4t^2 + 3t)\mathbf{j} + (-6t + 9)\mathbf{k} \text{ m} \\
 \therefore \mathbf{v} = \dot{\mathbf{r}} &= (6t + 4)\mathbf{i} + (-8t + 3)\mathbf{j} - 6\mathbf{k} \text{ m/s} \quad \blacklozenge \\
 \therefore \mathbf{a} = \dot{\mathbf{v}} &= 6\mathbf{i} - 8\mathbf{j} \text{ m/s}^2 \quad \blacklozenge
 \end{aligned}$$

(b) The vector normal to the plane formed by  $\mathbf{v}$  and  $\mathbf{a}$  (the instantaneous plane of motion) is

$$\mathbf{v} \times \mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6t + 4 & -8t + 3 & -6 \\ 6 & -8 & 0 \end{vmatrix} = -48\mathbf{i} - 36\mathbf{j} - 50\mathbf{k}$$

and the corresponding unit vector is

$$\mathbf{n} = \pm \frac{48\mathbf{i} + 36\mathbf{j} + 50\mathbf{k}}{\sqrt{48^2 + 36^2 + 50^2}} = \pm(0.615\mathbf{i} + 0.461\mathbf{j} + 0.640\mathbf{k})$$

Since this vector is independent of  $t$ , the orientation of the plane does not vary with the location of the particle. Thus the particle is in plane motion on an inclined plane. Q.E.D.

12.21

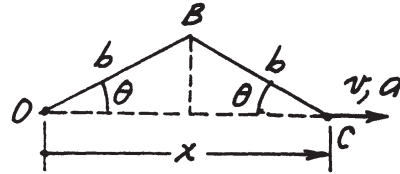
$$\begin{aligned}
 x &= R \cos \omega t & y &= R \sin \omega t & z &= \frac{R}{2} \sin 2\omega t \\
 \therefore v_x = \dot{x} &= -R\omega \sin \omega t & v_y = \dot{y} &= R\omega \cos \omega t & v_z = \dot{z} &= R\omega \cos 2\omega t \\
 \therefore v &= \sqrt{v_x^2 + v_y^2 + v_z^2} = R\omega \sqrt{\cos^2 \omega t + \sin^2 \omega t + \cos^2 2\omega t} = R\omega \sqrt{1 + \cos^2 2\omega t} \\
 \therefore v_{\max} &= \sqrt{2} R\omega \quad \blacklozenge \\
 a_x = \ddot{x} &= -R\omega^2 \cos \omega t & a_y = \ddot{y} &= -R\omega^2 \sin \omega t & a_z = \ddot{z} &= -2R\omega^2 \sin 2\omega t \\
 \therefore a &= \sqrt{a_x^2 + a_y^2 + a_z^2} = R\omega^2 \sqrt{\cos^2 \omega t + \sin^2 \omega t + 4 \sin^2 2\omega t} = R\omega^2 \sqrt{1 + 4 \sin^2 2\omega t} \\
 \therefore a_{\max} &= \sqrt{5} R\omega^2 \quad \blacklozenge
 \end{aligned}$$

12.22

(a) From geometry:  $x = 2b \cos \theta$

$$\therefore v = \dot{x} = -2b\dot{\theta} \sin \theta \rightarrow \blacklozenge$$

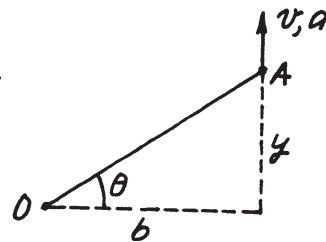
(b)  $\therefore a = \dot{v} = -2b(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta) \rightarrow \blacklozenge$



12.23

(a) Geometry:  $y = b \tan \theta \quad \therefore v = \dot{y} = b\dot{\theta} \sec^2 \theta \uparrow \blacklozenge$

$$\begin{aligned}
 \text{(b) } \therefore a &= \dot{v} = b[\ddot{\theta} \sec^2 \theta + 2\dot{\theta} \sec \theta (\sec \theta \tan \theta \dot{\theta})] \\
 &= b \sec^2 \theta (\ddot{\theta} + 2\dot{\theta}^2 \tan \theta) \uparrow \blacklozenge
 \end{aligned}$$



12.24

$$\begin{aligned}
 x &= R \cos \theta & v_x = \dot{x} &= (-R \sin \theta) \dot{\theta} \\
 y &= R \sin \theta & v_y = \dot{y} &= (R \cos \theta) \dot{\theta} \\
 v_y = v_0 & \text{ yields } \dot{\theta} &= \frac{v_0}{R \cos \theta} \\
 \therefore v_x &= (-R \sin \theta) \frac{v_0}{R \cos \theta} = -v_0 \tan \theta \\
 a_x = \dot{v}_x &= (-v_0 \sec^2 \theta) \dot{\theta} = (-v_0 \sec^2 \theta) \frac{v_0}{R \cos \theta} = -\frac{v_0^2}{R} \sec^3 \theta
 \end{aligned}$$

With  $R = 6$  m,  $v_0 = 2.5$  m/s and  $\theta = 60^\circ$  we get

$$\begin{aligned}
 v_y &= 2.5 \text{ m/s} & v_x &= -2.5 \tan 60^\circ = -4.330 \text{ m/s} \\
 \mathbf{v} &= -4.33\mathbf{i} + 2.5\mathbf{j} \text{ m/s} \quad \blacktriangleleft
 \end{aligned}$$

$$\begin{aligned}
 a_y &= 0 & a_x &= -\frac{2.5^2}{6} \sec^3 60^\circ = -8.333 \text{ m/s}^2 \\
 \mathbf{a} &= -8.33\mathbf{i} \text{ m/s}^2 \quad \blacktriangleleft
 \end{aligned}$$

12.25

$$\dot{\theta} = \frac{1200 \text{ rev}}{1.0 \text{ min}} \times \frac{2\pi \text{ rad}}{1.0 \text{ rev}} \times \frac{1.0 \text{ min}}{60 \text{ s}} = 125.66 \text{ rad/s}$$

$$r = 55 + 10 \cos \theta + 5 \cos 2\theta \text{ mm}$$

$$v = \dot{r} = \frac{dr}{d\theta} \dot{\theta} = (-10 \sin \theta - 10 \sin 2\theta)(125.66) \text{ mm/s}$$

$$a = \dot{v} = \frac{dv}{d\theta} \dot{\theta} = (-10 \cos \theta - 20 \cos 2\theta)(125.66)^2 \text{ mm/s}^2$$

$$|a|_{\text{max}} = 30(125.66)^2 = 474\,000 \text{ mm/s}^2 = 474 \text{ m/s}^2 \text{ (at } \theta = 0) \blacktriangleleft$$

\*12.26

(a) Geometry:  $x = y \tan \theta_A = y \tan \theta_B + b \dots\dots\dots$  (a)

$$\therefore y = \frac{b}{\tan \theta_A - \tan \theta_B} = \frac{1000}{\tan 30^\circ - \tan 22^\circ} = 5770 \text{ m} \blacklozenge$$

(b) Differentiate Eq. (a):

$$\dot{x} = y \dot{\theta}_A \sec^2 \theta_A + \dot{y} \tan \theta_A = y \dot{\theta}_B \sec^2 \theta_B + \dot{y} \tan \theta_B \text{ (b)}$$

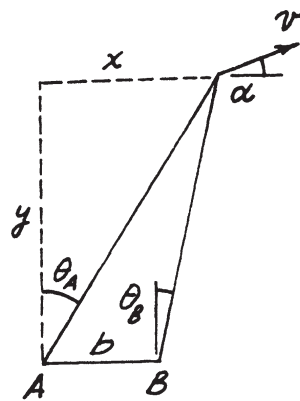
$$\therefore \dot{y} = -y \frac{\dot{\theta}_A \sec^2 \theta_A - \dot{\theta}_B \sec^2 \theta_B}{\tan \theta_A - \tan \theta_B}$$

$$= -(5770) \frac{(0.026) \sec^2 30^\circ - (0.032) \sec^2 22^\circ}{\tan 30^\circ - \tan 22^\circ} = 85.11 \text{ m/s}$$

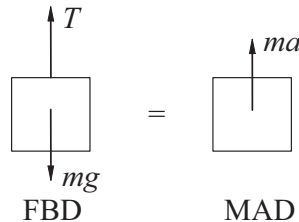
$$\text{From Eq. (b): } \dot{x} = (5770)(0.026) \sec^2 30^\circ + (85.11) \tan 30^\circ = 249.2 \text{ m/s}$$

$$\therefore v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{249.2^2 + 85.11^2} = 263 \text{ m/s} \blacklozenge$$

$$\text{(c) } \therefore \alpha = \tan^{-1}(\dot{y}/\dot{x}) = \tan^{-1} \frac{85.11}{249.2} = 18.86^\circ \blacklozenge$$



12.27

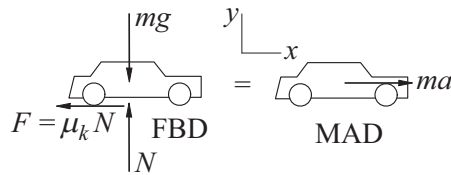


$$v = 4t \text{ m/s} \quad a = \dot{v} = 4 \text{ m/s}^2$$

$$\Sigma F = ma \quad + \uparrow \quad T - mg = ma$$

$$T = m(g + a) = 50(9.81 + 4) = 691 \text{ N} \blacktriangleleft$$

12.28



$$v_0 = 100 \text{ km/h} = 100 \text{ km/h} \left( \frac{1000 \text{ m/km}}{3600 \text{ s/h}} \right) = 27.78 \text{ m/s}$$

$$\Sigma F_y = 0 \quad + \uparrow \quad N - mg = 0 \quad \therefore N = mg$$

$$\Sigma F_x = ma \quad \rightarrow \quad -\mu_k N = ma \quad \therefore a = -\frac{\mu_k N}{m} = -\mu_k g$$

$$v = \int a \, dt = -\mu_k g t + C_1$$

$$x = \int v \, dt = -\frac{1}{2} \mu_k g t^2 + C_1 t + C_2$$

When  $t = 0$  (initial conditions):

$$x = 0 \quad \therefore C_2 = 0 \quad v = v_0 \quad \therefore C_1 = v_0$$

$$\therefore x = -\frac{1}{2} \mu_k g t^2 + v_0 t \quad v = -\mu_k g t + v_0$$

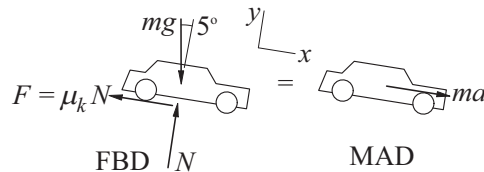
When  $v = 0$ :

$$-\mu_k g t + v_0 \quad \therefore t = \frac{v_0}{\mu_k g}$$

$$x = -\frac{1}{2} \mu_k g \left( \frac{v_0}{\mu_k g} \right)^2 + v_0 \left( \frac{v_0}{\mu_k g} \right) = \frac{v_0^2}{2 \mu_k g}$$

$$= \frac{27.78^2}{2(0.65)(9.81)} = 60.5 \text{ m} \quad \blacktriangleleft$$

12.29



$$v_0 = 100 \text{ km/h} = 27.78 \text{ m/s}$$

$$\begin{aligned}\Sigma F_y = 0 \quad + \uparrow \quad N - mg \cos 5^\circ = 0 \quad \therefore N = mg \cos 5^\circ \\ \Sigma F_x = ma \quad \rightarrow \quad -\mu_k N + mg \sin 5^\circ = ma \\ \therefore a = -\frac{\mu_k N}{m} + g \sin 5^\circ = (\sin 5^\circ - \mu_k \cos 5^\circ)g \\ = (\sin 5^\circ - 0.65 \cos 5^\circ)9.81 = -5.497 \text{ m/s}^2\end{aligned}$$

$$\begin{aligned}v &= \int a \, dt = -5.497t + C_1 \\ x &= \int v \, dt = -2.749t^2 + C_1t + C_2\end{aligned}$$

When  $t = 0$  (initial conditions):

$$x = 0 \quad \therefore C_2 = 0 \quad v = v_0 \quad \therefore C_1 = v_0 = 27.78 \text{ m/s}$$

$$\begin{aligned}x &= -2.749t^2 + 27.78t \text{ m} \\ v &= -5.497t + 27.78 \text{ m/s}\end{aligned}$$

When  $v = 0$ :

$$\begin{aligned}-5.497t + 27.78 = 0 \quad \therefore t = 5.054 \text{ s} \\ x = -2.749(5.054)^2 + 27.78(5.054) = 70.2 \text{ m} \quad \blacktriangleleft\end{aligned}$$

### 12.30

$$\begin{aligned}a &= \frac{F}{m} = \frac{-1.2t}{0.1} = -12t \text{ m/s}^2 \\ v &= \int a_x dt = -6t^2 + C_1 \text{ m/s} \\ x &= \int v_x dt = -2t^3 + C_1t + C_2 \text{ m}\end{aligned}$$

When  $t = 0$  (initial conditions):

$$\begin{aligned}x = 0 \quad \therefore C_2 = 0 \quad v = 64 \text{ m/s} \quad \therefore C_1 = 64 \text{ m/s} \\ \therefore x = -2t^3 + 64t \text{ m} \quad v = -6t^2 + 64 \text{ m/s}\end{aligned}$$

When  $t = 4$  s:

$$x = -2(4)^3 + 64(4) = 128 \text{ m}$$

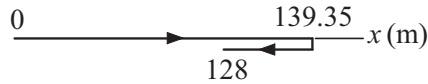
When  $v = 0$ :

$$\begin{aligned}-6t^2 + 64 = 0 \quad t = 3.266 \text{ s} \\ x = -2(3.266)^3 + 64(3.266) = 139.35 \text{ m}\end{aligned}$$



Distance traveled:

$$d = 2(139.35) - 128 = 150.7 \text{ m} \blacktriangleleft$$



### 12.31

$$a = \frac{F}{m} = \frac{0.06\sqrt{v}}{0.012} = 5\sqrt{v} \text{ m/s}^2$$

$$dt = \frac{dv}{a} = \frac{dv}{5\sqrt{v}} \quad t = \int \frac{dv}{5\sqrt{v}} = \frac{2}{5}\sqrt{v} + C_1$$

Given  $v = 0.25 \text{ m/s}$  when  $t = 0.8 \text{ s}$ :

$$0.8 = \frac{2}{5}\sqrt{0.25} + C_1 \quad C_1 = 0.6 \text{ s}$$

$$t = \frac{2}{5}\sqrt{v} + 0.6 \text{ s}$$

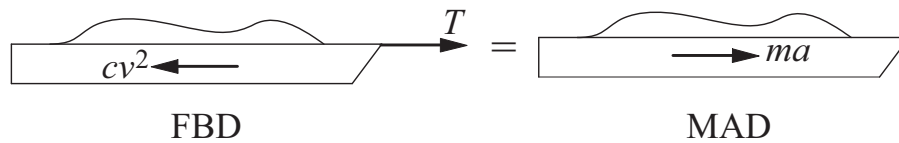
$$v = (2.5t - 1.5)^2 = 6.25t^2 - 7.5t + 2.25 \text{ m/s}$$

$$x = \int v \, dt = \frac{6.25}{3}t^3 - \frac{7.5}{2}t^2 + 2.25t + C_2$$

Initial condition:  $x = 0$  when  $t = 0 \quad \therefore C_2 = 0$

$$x|_{t=1.2\text{s}} = \frac{6.25}{3}(1.2)^3 - \frac{7.5}{2}(1.2)^2 + 2.25(1.2) = 0.90 \text{ m} \blacktriangleleft$$

### 12.32



$$ma = T - F_D = T - cv^2 \quad a = \frac{T - cv^2}{m}$$

$$x = \int \frac{v}{a} dv = m \int \frac{v}{T - cv^2} dv + C = -\frac{m}{2c} \ln(T - cv^2) + C$$

Initial condition:  $v = 0$  at  $x = 0$ :

$$0 = -\frac{m}{2c} \ln(T) + C \quad C = \frac{m}{2c} \ln(T)$$

$$\therefore x = -\frac{m}{2c} \ln(T - cv^2) + \frac{m}{2c} \ln(T) = \frac{m}{2c} \ln \frac{T}{(T - cv^2)}$$

Solve for  $v$ :

$$\frac{T}{(T - cv^2)} = \exp\left(\frac{2c}{m}x\right) \quad T - cv^2 = T \exp\left(-\frac{2c}{m}x\right)$$

$$v^2 = \frac{T}{c} \left[1 - \exp\left(-\frac{2c}{m}x\right)\right] \quad v = \sqrt{\frac{T}{c} \left[1 - \exp\left(-\frac{2c}{m}x\right)\right]} \blacktriangleleft$$

Terminal velocity:

$$v_\infty = \lim_{x \rightarrow \infty} v(x) = \sqrt{\frac{T}{c}} \blacktriangleleft$$

### 12.33

$$a = \frac{F}{m} = \frac{4t - 4}{4} = t - 1 \text{ m/s}^2$$

$$v = \int a \, dt = \frac{1}{2}t^2 - t + C_1$$

$$y = \int v \, dt = \frac{1}{6}t^3 - \frac{1}{2}t^2 + C_1t + C_2$$

When  $t = 0$  (initial condition):

$$y = 0 \quad \therefore C_2 = 0 \quad v = -8 \text{ m/s} \quad \therefore C_1 = -8 \text{ m/s}$$

$$\therefore y = \frac{1}{6}t^3 - \frac{1}{2}t^2 - 8t \text{ m} \quad v = \frac{1}{2}t^2 - t - 8 \text{ m/s}$$

When  $t = 8$  s:

$$y = \frac{1}{6}(8)^3 - \frac{1}{2}(8)^2 - 8(8) = -10.67 \text{ m}$$

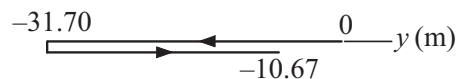
When  $v = 0$ :

$$\frac{1}{2}t^2 - t - 8 = 0 \quad t = 5.123 \text{ s}$$

$$y = \frac{1}{6}(5.123)^3 - \frac{1}{2}(5.123)^2 - 8(5.123) = -31.70 \text{ m}$$

Distance traveled:

$$d = 2(31.70) - 10.67 = 52.7 \text{ m} \blacktriangleleft$$

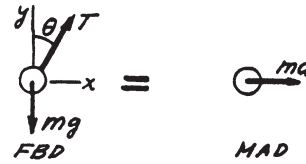


12.34

$$\Sigma F_y = ma_y: +\uparrow T \cos\theta - mg = 0$$

$$\Sigma F_x = ma_x: \rightarrow T \sin\theta = ma$$

The solution is:  $\theta = \tan^{-1} \frac{a}{g}$  ♦



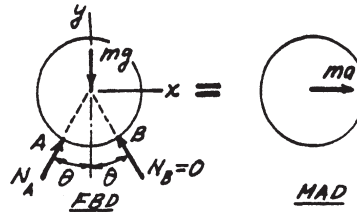
12.35

Assume impending climbing ( $N_B = 0$ ).

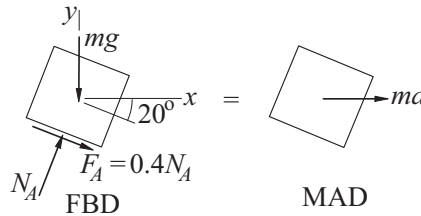
$$\Sigma F_y = ma_y: +\uparrow N_A \cos\theta - mg = 0$$

$$\Sigma F_x = ma_x: \rightarrow N_A \sin\theta = ma$$

The solution is:  $a = g \tan\theta$  ♦



12.36



Assume impending sliding ( $F_A = 0.4N_A$ )

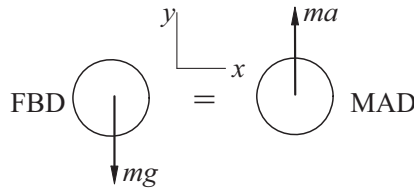
$$\Sigma F_y = 0 + \uparrow N_A \cos 20^\circ - 0.4N_A \sin 20^\circ - mg = 0$$

$$N_A = 1.2455 mg$$

$$\Sigma F_x = ma_x \rightarrow N_A \sin 20^\circ + 0.4N_A \cos 20^\circ = ma$$

$$1.2455 mg(\sin 20^\circ + 0.4 \cos 20^\circ) = ma \quad a = 0.894g \blacktriangleleft$$

12.37 Let  $y$  be measured up from the base of the cliff.



$$\Sigma F_y = ma + \uparrow -mg = ma \quad \therefore a = -g$$

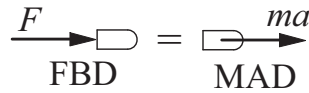
$$a = \frac{dv}{dy}v \quad \therefore v dv = -g dy \quad \frac{1}{2}v^2 = -gy + C$$

Initial condition:  $v = v_0$  when  $y = h$ .  $\therefore C = \frac{1}{2}v_0^2 + gh$

$$\therefore \frac{1}{2}(v^2 - v_0^2) = g(h - y)$$

At impact  $y = 0$   $\therefore \frac{1}{2}(v^2 - v_0^2) = gh$   $\therefore v = \sqrt{v_0^2 + 2gh}$  ◀

12.38



$$F = ma \quad a = \frac{F}{m} = \frac{F_0}{m}e^{-x/b} \quad v dv = \frac{F_0}{m}e^{-x/b}dx$$

$$\frac{1}{2}v^2 = \frac{F_0}{m} \int e^{-x/b} dx = -\frac{F_0 b}{m}e^{-x/b} + C$$

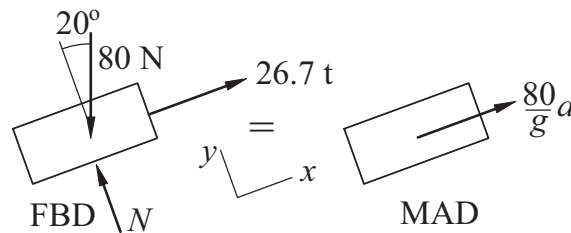
Initial condition :  $v = 0$  at  $x = 0$   $\therefore C = \frac{F_0 b}{m}$

$$\frac{1}{2}v^2 = \frac{F_0 b}{m} (1 - e^{-x/b})$$

$$v = \sqrt{\frac{2F_0 b}{m} (1 - e^{-x/b})}$$

$$v|_{x=0.55 \text{ m}} = \sqrt{\frac{2(7010 \text{ N})(2)}{0.0275/9.81} (1 - e^{-0.55/0.61})} = 1346 \text{ m/s} \quad \blacktriangleleft$$

12.39



$$\Sigma F_x = ma \quad 26.7t - 80 \sin 20^\circ = \frac{80}{9.81}a$$

$$a = \frac{9.81}{80}(26.7t - 80 \sin 20^\circ) = 3.274t - 3.355 \text{ m/s}^2$$

$$v = \int a dt = 1.637t^2 - 3.355t + C_1 \text{ m/s}$$

$$x = \int v dt = 0.5457t^3 - 1.678t^2 + C_1t + C_2 \text{ m}$$