2 A Telecommunication System

2.3

In Figure 2.2 in the textbook, we note that $W(f)$ and $S(f)$ have no frequency content in common. Consequently, all of the frequencies present in $W(f)$ are not in $S(f)$, and vice versa. In other words, the frequencies present in $W(f)$ but not $S(f)$ are all f such that $|f| < f^\dagger$. And the frequencies present in $S(f)$ but not $W(f)$ are all f such that $-f_0 - f^\dagger < f < -f_0 + f^\dagger$ and $f_0 - f^\dagger < f < f_0 + f^\dagger$.

2.4

Denote $W(f) = A(f)e^{-j\phi(f)}$, then from equation (2.5), we have

$$\begin{aligned} S(f) &= \frac{1}{2}W(f-f_0) + \frac{1}{2}W(f+f_0) \\ &= \frac{1}{2}A(f-f_0)e^{-j\phi(f-f_0)} + \frac{1}{2}A(f+f_0)e^{-j\phi(f+f_0)}. \end{aligned}$$

The phase spectrum of $s(t)$ is the shifted version of the phase spectrum of $w(t)$. The picture would look very similar to Figure 2.2 in the textbook.

2.5

Start with the equation (2.5):

$$\begin{aligned} R(f) = F\{r(t)\} &= F\{s(t)\cos(2\pi f_1 t)\} \\ &= F\{s(t)[\frac{1}{2}(e^{j2\pi f_1 t} + e^{-j2\pi f_1 t})]\} \\ &= \int_{-\infty}^{+\infty} s(t)[\frac{1}{2}(e^{j2\pi f_1 t} + e^{-j2\pi f_1 t})]e^{-j2\pi f t}dt \\ &= \frac{1}{2}\int_{-\infty}^{+\infty} s(t)e^{-j2\pi(f-f_1)t}dt + \frac{1}{2}\int_{-\infty}^{+\infty} s(t)e^{-j2\pi(f_1+f)t}dt \\ &= \frac{1}{2}S(f-f_1) + \frac{1}{2}S(f+f_1). \end{aligned}$$

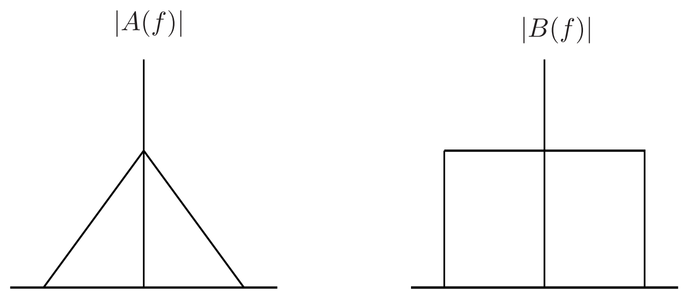


Figure 2.1: Exercise 2.6 Bandlimited baseband signals

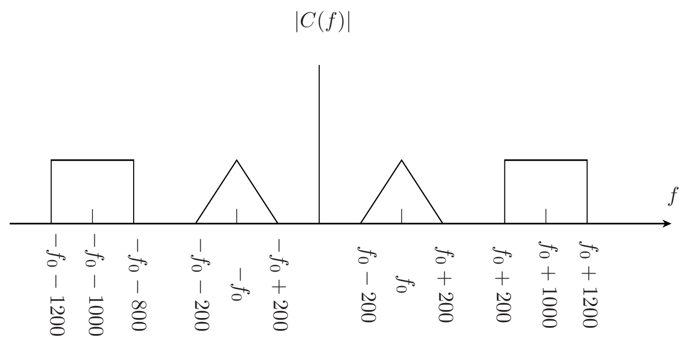


Figure 2.2: Exercise 2.6 Baseband bandwidth of 200 Hz, modulated with 1 kHz carrier

Since $S(f) = \frac{1}{2}W(f - f_0) + \frac{1}{2}W(f + f_0)$ (from (2.5)), this yields the final answer:

$$\begin{aligned} R(f) &= \frac{1}{2}S(f - f_1) + \frac{1}{2}S(f + f_1) \\ &= \frac{1}{4}W(f - f_1 - f_0) + \frac{1}{4}W(f - f_1 + f_0) + \frac{1}{4}W(f + f_1 - f_0) + \frac{1}{4}W(f + f_1 + f_0) \end{aligned}$$

2.6

Take any two baseband signals, bandlimited to the same frequency as listed in Fig. 2.1.

(a) If the signals are modulated by 1kHz separated carriers and are bandlimited to 200 Hz, the spectrum of the signals after modulation are shown in Fig. 2.2.
(b) If the signals now have a bandwidth of 2 kHz and are modulated by the same carriers, the spectrum of the signals after modulation are shown in Fig. 2.3 (note: overlapped areas actually add together).

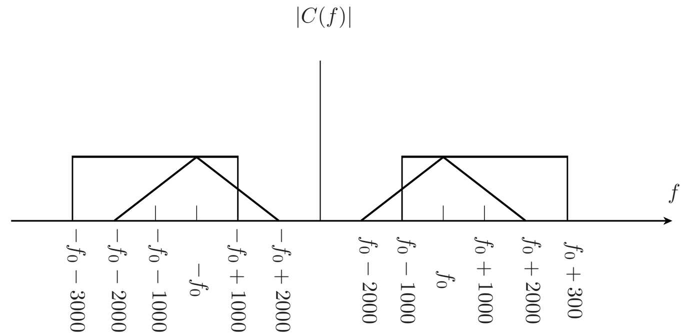


Figure 2.3: Exercise 2.6 Baseband bandwidth of 2 kHz, modulated with two carrier frequencies separated by 1 kHz

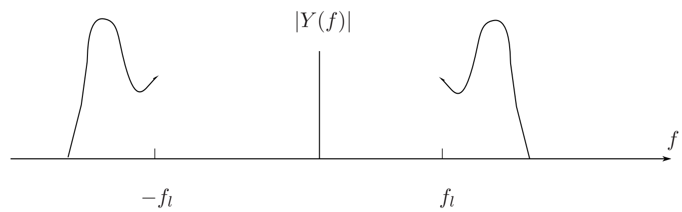


Figure 2.4: Exercise 2.7 TB Figure 2.5 passed through a HPF

The signals in part (a) do not alias and thus can be recovered separately with bandpass filters at the receiver. The signals alias in part (b) so they can not be extracted with bandpass filters and the originals can not be recovered at the receiver.

2.7

If the signal in Figure 2.5 of the textbook is put through an ideal highpass filter with cutoff f_l its spectrum will look like as in Fig. 2.4.

2.8

If the signal in Figure 2.5 of the textbook is put through an ideal bandpass filter with the specified cutoffs, its spectrum will look like as shown Fig. 2.5.

2.9

(a) Recall that things which are narrow in time are wider in frequency. Since $p_2(t)$ is the summation of two pulses (one delayed) of width $T/2$, $p_2(t)$ must occupy more bandwidth than $p_1(t)$. Note also that $p_3(t)$ is the convolution (in time) of

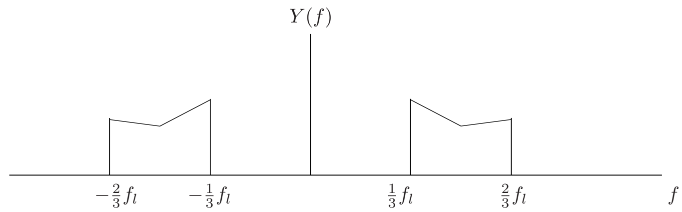


Figure 2.5: Exercise 2.8 TB Figure 2.5 passed through a LPF

two pulses of width $T/2$, so it has larger bandwidth than $p_1(t)$. However, self-convolution in time corresponds to squaring in the frequency domain. Squaring reduces the width of the frequency domain representation, so the frequency domain representation of $p_3(t)$ is narrower than $p_2(t)$. Answer: $p_2(t)$.

(b) From part (a), we know that $p_2(t)$ occupies the largest bandwidth. It thus remains to compare $p_1(t)$ and $p_3(t)$. Either answer is accepted since there is no easy way to compare the bandwidth of these two pulses given what we have learned in the course to date.

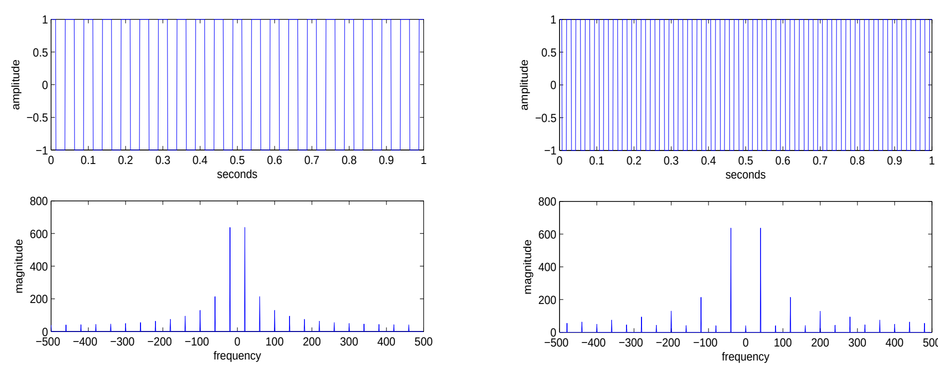
3 The Six Elements

3.1

(a) The plots are as shown in Fig. 3.1. As the frequency increases, the number of cycles increases, so the time domain plots squeeze together. Inversely, in the frequency domain, the harmonic frequency spikes spread out. This is a standard property of the Fourier transform. Additionally, as the frequency increases, the aliasing in the frequency domain becomes more noticeable. The Fourier series representation of a square wave has an infinite number of odd harmonic coefficients, so there is aliasing in all the spectra, but it is much more noticeable when the frequency is larger.

(b) As the time variable increases, the magnitude of the spikes in the spectra increase proportionally. With a time of 1, the spikes have a magnitude of 600. At a time of two, the amplitude doubles to 1200. With a time of 10, the amplitude becomes 6000. The plots for a square wave of duration 100 are shown in Fig. 3.2.

(c) As the sampling rate decreases (T_s increases), the quality of the signal construction decreases. If the sampling rate becomes too low, aliasing significantly



(a) Frequency of 20 Hz (b)Frequency of 40 Hz

Figure 3.1: Exercise 3.1 Plot of square wave and its spectrum at different frequencies

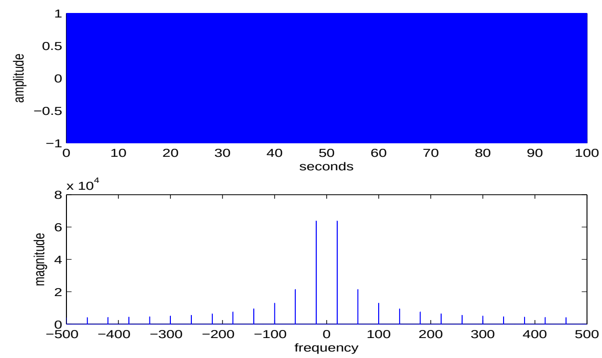


Figure 3.2: Exercise 3.1(b) Plot of square wave with time = 100s, f = 20Hz and its spectrum

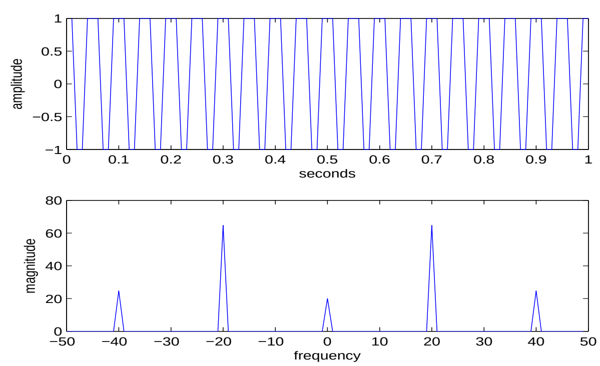


Figure 3.3: Exercise 3.1(c) Plot of square wave with $T_s = 1/100$ and its spectrum

alters the signal. As the sampling rate increases, the signals look smoother and more continuous. This is best seen by comparing Fig. 3.1(a) and Fig. 3.3:

3.3

(a) $s(t) = e^{-t}$, as shown in Fig. 3.4.
(b) $s(t) = 5e^{-t}$, as shown in Fig. 3.5.
(c) $s(t) = e^{-t^2}$ for $-2 \leq t \leq 2$, as shown in Fig. 3.6.
(d) $s(t) = e^{-t^2}$ for $-20 \leq t \leq 20$, as shown in Fig. 3.7.
(e) $s(t) = \sin(2\pi f t + \phi)$ for $0 \leq t \leq 2$, as shown in Fig. 3.8 and Fig. 3.9.

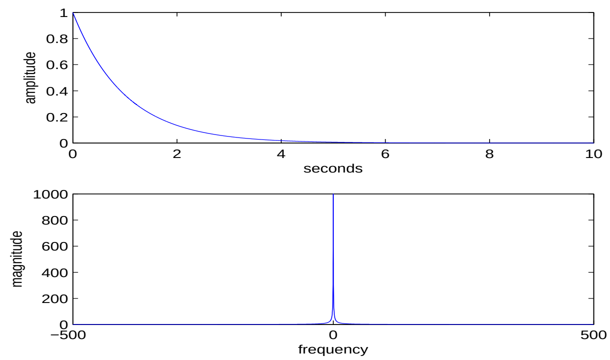


Figure 3.4: Exercise 3.3(a) Spectrum of $s(t) = e^{-t}$

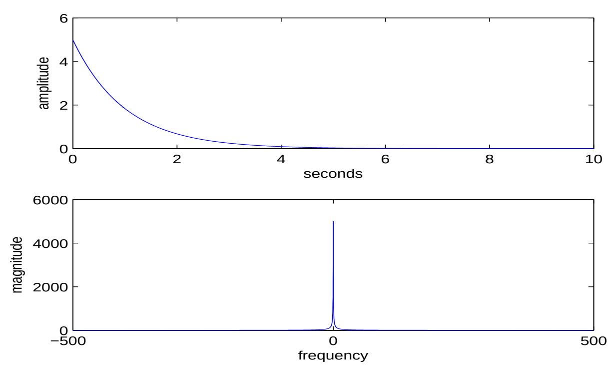


Figure 3.5: Exercise 3.3(b) Spectrum of $s(t) = 5e^{-t}$

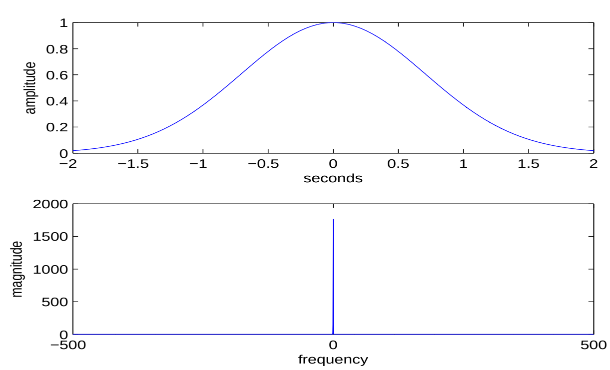


Figure 3.6: Exercise 3.3(c) Spectrum of $s(t) = e^{-t^2}$ for $-2 \leq t \leq 2$

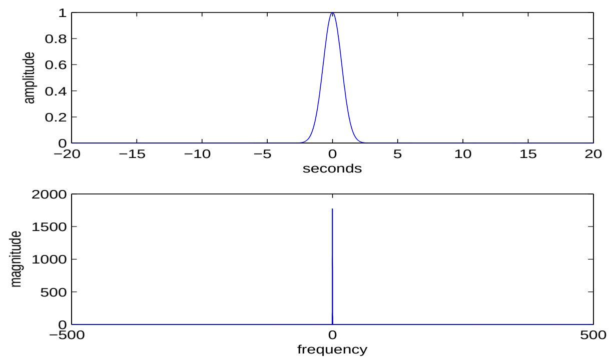
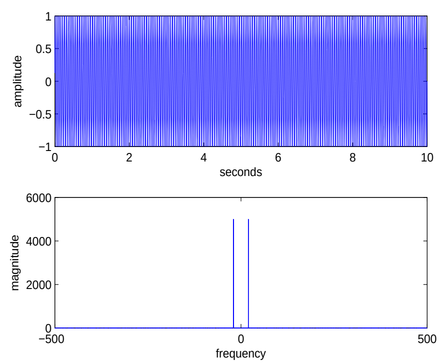
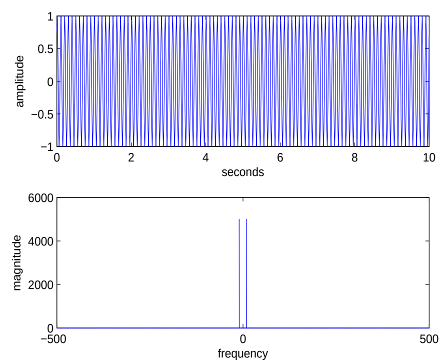


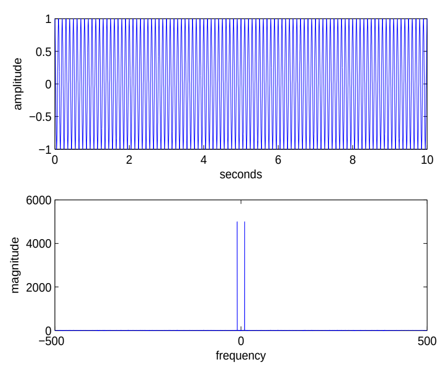
Figure 3.7: Exercise 3.3(d) Spectrum of $s(t) = e^{-t^2}$ for $-20 \leq t \leq 20$



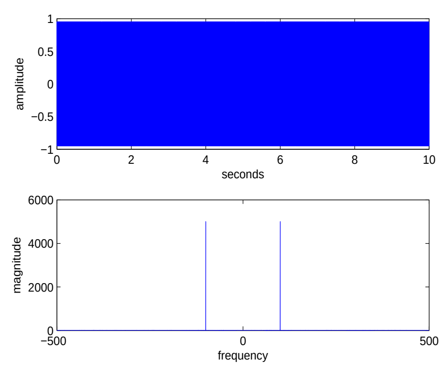
$f = 20, \phi = 0$



$f = 20, \phi = \pi/4$

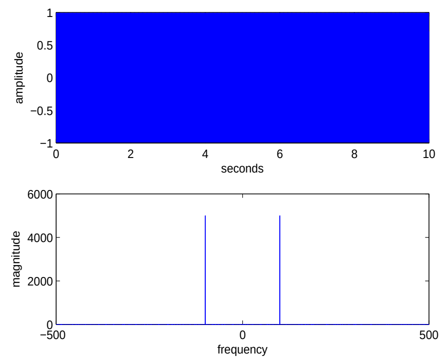


$f = 20, \phi = \pi/2$

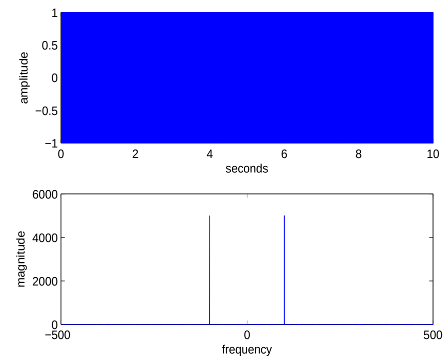


$f = 100, \phi = 0$

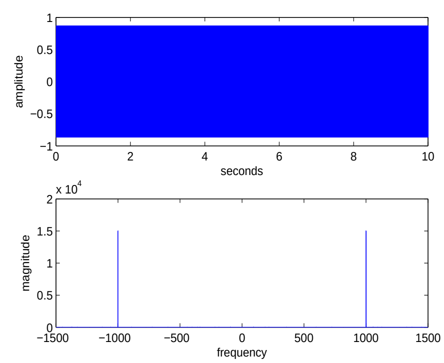
Figure 3.8: Exercise 3.3(e) Spectrum of $s(t) = \sin(2\pi f t + \phi)$ for various f and ϕ



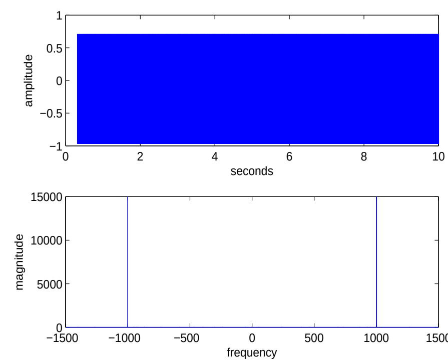
$f = 100, \phi = \pi/4$



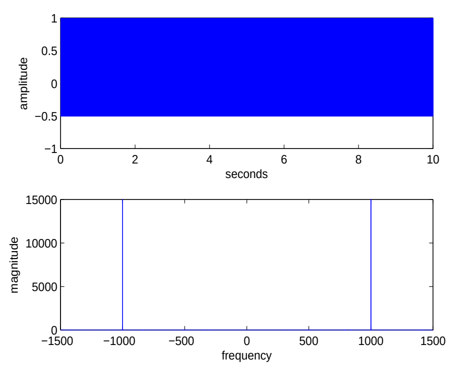
$f = 100, \phi = \pi/2$



$f = 1000, \phi = 0$



$f = 1000, \phi = \pi/4$



$f = 1000, \phi = \pi/2$

Figure 3.9: Exercise 3.3(e) Spectrum of $s(t) = \sin(2\pi f t + \phi)$ for various f and ϕ (Cont'd)

3.4

(a) The spectrum of uniformly distributed noise on the interval [-1,1] is shown in Fig. 3.10.
(b) The spectrum of the signal is shown in Fig. 3.11.
(c) The spectrum of the signal is shown in Fig. 3.12.

3.6

(a) Plots for various frequencies are shown in Fig. 3.13.
(b) Plots for various phases are shown in Fig. 3.14.
(c) Plots for various sampling rates are in Fig. 3.15.

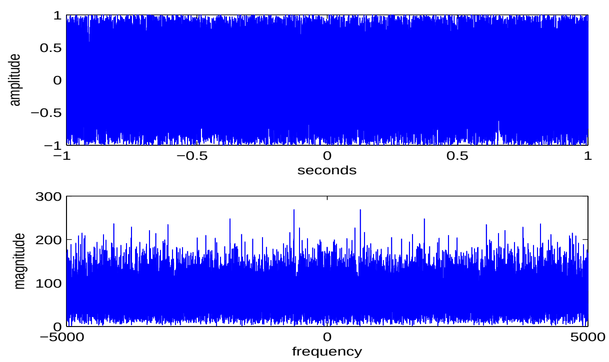


Figure 3.10: Exercise 3.4(a) Uniform noise spectrum

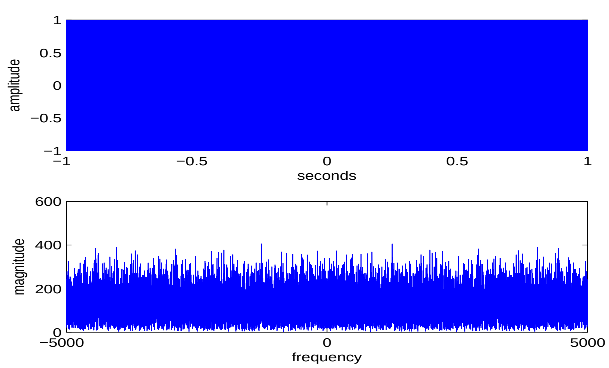


Figure 3.11: Exercise 3.4(b) Spectrum of +1, -1 signal