Chapter 1

Introduction

Section 1.1

1.1 Design transport containers for milk in 1 gallon and 1 liter sizes.

Notes: There are a number of design considerations that can be included in this problem. Some examples are considered here, but the student should not be restricted in the types of solutions pursued. This type of problem is an excellent opportunity for brainstorming, where a group is encouraged to provide all possible answers to the problem regardless of feasibility. For example, the materials considered could include wood barrels, ceramic jugs or foil or plastic bags, as in intraveneous fluid containers.

Solution: The transport containers for milk should be easy to handle and store as well as being easy to pour from without spilling. The material should be recyclable or be able to be burned leaving harmless combustion products. The obvious material choices are paper or plastic materials, although glass bottles and metal cans can be considered as possibilities. For a one liter container the standard type Pure Pac or Tetra Pac Bric made of plastic carton is a good design. A one gallon container has to be compact to be easily stored in a refrigerator. The width, depth, and height should be similar in size, which would be about 156 mm x 156 mm x 156 mm. That size is too large to grip with one hand. Therefore, a one gallon container is preferably made of a plastic with an integral handle.

1.2 Design a kit of tools for campers so they can prepare and eat meals. The kit should have all of the implements needed, be lightweight and compact.

Notes: See the notes to problem 1.

Solution: If they are not hung simultaneously, a steel wire frame of triangular shape with a hook at the top could be used. The horizontal portion could be covered with a high friction coating or tape or glue to keep the trousers from sliding off. For coats, a sturdier frame is needed, requiring either a larger gage of wire or pursuit of another alternative such as wood or plastic hangers.

Section 1.3

1.3 Journal bearings on train boxcars in the early 19th century used a stink additive in their lubricant. If the bearing got too hot, it would attain a noticeable odor, and an oiler would give the bearing a squirt of lubricant at the next train stop. What design philosophy does this illustrate? Explain.

Solution: The most obvious design philosophy is the Doctrine of Manifest Danger. This philosophy suggests that systems be designed so that a failure can be detected before it occurs. In this case, a dry bearing becomes hot, and the bad smell indicates a loss of lubricant before the bearing failure is catastrophic. It should be noted that other answers are possible. For example, the principal of uniform safety suggests that the level of safety of all components be kept at the same level. If failure of one component, in this case a bearing, is imminent, then extra maintenance on this bearing is justified by the principle of uniform safety.

1.4 Explain why engineers must work with other disciplines, using specific product examples.

Solution: Modern product design has become a multidisciplinary endeavor. It is essential that engineers be able to work with other disciplines. For example:

(a) Engineers who design orthopedic implants must be able to speak with surgeons in order to produce designs that work in the human body.

(b) Most consumer electronics products require engineers to work with marketing persons in order to develop an attractive package and a sales strategy.

(c) Design of products with large liability exposure, such as infant car seats, require communication with attorneys and safety specialists to make sure the product is suitable.

Section 1.4

1.5 A hand-held drilling machine has a bearing to take up radial and thrust load from the drill. Depending on the number of hours the drill is expected to be used before it is scrapped, different bearing arrangements will be chosen. A rubbing bushing has a 50-hr life. A small ball bearing has a 300-hr life. A two-bearing combination of a ball bearing and a cylindrical roller bearing has a 10,000-hr life. The cost ratios for the bearing arrangements are 1:5:20. What is the optimum bearing type for a simple drill, a semiprofessional drill, and a professional drill?

Notes: This problem is fairly subjective, but there should be a realization that there is an increase in expected life for a professional drill versus a simple drill. In fact, for certain users, a 50 hour life is probably sufficient.

Solution: Notice that the cost per time is highest for the rubbing bearing and lowest for the ball bearing cylindrical bearing combination. However, the absolute cost is higher for the longer lasting components, so the expected life must be assessed. Rough estimates based on expected lives are as follows.

(a) Simple drill - use rubbing bearings.

(b) Semi-professional drill - either two ball bearings or a ball and cylindrical roller bearing combination.

(c) Professional drill - use a ball and cylindrical roller bearing combination.

1.6 Using the hand-held drill described in Problem 1.5, if the solution with the small ball bearing was chosen for a semiprofessional drill, the bearing life could be estimated to be 300 hr until the first spall forms in the race. The time from first spall to when the whole rolling-contact surface is covered with spalls is 200 hr, and the time from then until a ball cracks is 100 hr. What is the bearing life

(a) If high precision is required?

(b) If vibrations are irrelevant?

(c) If an accident can happen when a ball breaks?

Notes: This problem is fairly subjective, but there should be a realization that there is a decrease in expected life for the high precision drill, etc.

Solution:

(a) For high precision, no spall should be allowed so the life is 300 hrs.

(b) If vibrations are irrelevant, the drill can be used until a ball breaks, or 600hours.

(c) Depending on the severity of the accident, it is possible that an entirely different bearing should be used. However, safety allowances of 300-500 hours may be reasonable.

1.7 A car is being driven at 150 km/hr on a mountain road where the posted speed limit is 100 km/hr. At a tight turn, one of the tires fails (a blowout) causing the driver to lose control and results in an accident involving property losses and injuries but no loss of life. Afterwards, the driver decides to file a lawsuit against the tire manufacturer. Explain which legal theories give him a viable argument to make a claim.

Solution: Consider the following theories.

(a) *Caveat Emptor.* Let the buyer beware does not give the driver a cause for a lawsuit.

(b) *Negligence Theory.* There may be a cause of action based on negligence theory, but the driver is at risk since his actions will also be considered.

(c) *Strict Liability.* This may be the best chance for the driver, since the emphasis will be on the product, and whether or not a tire should allow travel at 150 km/hr without catastrophic failure.

1.8 The dimensions of skis used for downhill competition need to be determined. The maximum force transmitted from one foot to the ski is 2500 N, but the snow conditions are not known in advance, so the bending moment acting on the skis is not known. Estimate the safety factor needed.

Notes: There are two different approaches, that of section 1.5.1, and that of a worst case scenario.

Solution: Using the approach of Section 1.5.1, the largest safety factor from Table 1.1 is $n_{sx} = 3.95$, and the largest value of n_{sy} from Table 1.2 is 1.6. Therefore, the largest safety factor, if there is poor control of materials and loading, and if the consequences are sever, is given by Eq. (1.2) as:

$$n_s = n_{sx} n_{sy} = (3.95)(1.6) = 6.32$$

Another approach is to perform a worst case scenario analysis. If the weight of skier and equipment is 100 kg and the maximum load on the ski is 2500 N, then the centrifugal acceleration is 23 m/s^2. If the speed of the skier is 35 m/s (126 km/hr), the radius from the ski track will be 53 m. If the speed is 20 m/s, the radius is 17.4 m. This means that no overstressing should ever occur due to bending of the ski, so the safety factor could be chosen just above one.

1.9 A crane has a loading hook that is hanging in a steel wire. The allowable normal tensile stress in the wire gives an allowable force of 100,000 N. Estimate the safety factor that should be used.

(a) If the wire material is not controlled, the load can cause impact, and fastening the hook in the wire causes stress concentrations. (If the wire breaks, people can be seriously hurt and expensive equipment can be destroyed.)

(b) If the wire material is extremely well controlled, no impact loads are applied, and the hook is fastened in the wire without stress concentrations. (If the wire breaks, no people or expensive equipment can be damaged.)

Notes: Equation (1.2) is needed to solve this problem, with data obtained from Tables 1.1 and 1.2.

Solution:

(a) If the quality of materials is poor ($A = p$), the control over the load to the part is poor ($B = p$), and the accuracy of the stress analysis is poor ($C = p$), then $n_{sx} = 3.95$ as given in Table 1.1. If the economic impact and danger are both very serious ($D = E = vs$), then from Table 1.2, $n_{sy} = 1.6$. Therefore, the required safety factor is obtained from Eq. (1.2) as

$$n_s = n_{sx} n_{sy} = (3.95)(1.6) = 6.3$$

(b) If the quality of materials, the control over the load to the part, and the accuracy of the stress analysis are all very good ($A = B = C = vg$), then $n_{sx} = 1.1$ as given in Table 1.1. If the economic impact and danger are both not serious ($D = E = ns$), then from Table 1.2, $n_{sy} = 1.0$. Therefore, the required safety factor is obtained from Eq. (1.2) as

$$n_s = n_{sx}n_{sy} = (1.1)(1.0) = 1.1$$

1.10 Give three examples of fail-safe and three examples of fail-unsafe products.

Notes: This is an open-ended problem, and many different solutions are possible.

Solution: Examples of fail safe products are:

(a) electric motors, which fail to move and cause no hazards when they fail.

(b) Bowling pins, because of their distance from bowlers, do not present any hazards when they shatter.

(c) Furniture upholstery, which causes aesthetic objection long before structural failure occurs.

Examples of fail unsafe products include:

(a) Bungee cords, as in bungee cord jumping from bridges and the like;

(b) parachutes, which cause a fatal injury in the vast majority of failures, and

(c) A fan on an exhaust hood over a fume producing tank, such as an electroplating tank. Failure of the forced air circulation system exposes plant personnel to harmful fumes.

1.11 An acid container will damage the environment and people around it if it leaks. The cost of the container is proportional to the container wall thickness. The safety can be increased either by making the container wall thicker or by mounting a reserve tray under the container to collect the leaking acid. The reserve tray costs 10% of the thick-walled container cost. Which is less costly, to increase the wall thickness or to mount a reserve tray under the container?

Notes: This problem uses the approach described by Eq. (1.2) and Tables 1.1 and 1.2.

Solution: If the safety factor is unchanged, then for a thicker container wall, we see that with danger to personnel rated very serious and economic impact very serious (D=E=vs), then $n_{sy} = 1.6$. If a reserve tray is used, the required safety factor for the container (D=E=ns) is $n_{sy} = 1.0$. Therefore, if A is the cost of the container with thicker walls, then the cost of the container with a reserve tray is

$$\frac{A}{1.6} + (0.1)A = 0.725A$$

Therefore, with the same safety factor, it is less costly to have the reserve tray.

Section 1.7

1.12 Calculate the following:

(a) The velocity of hair growth in meters per second, assuming hair grows 0.75 in. in one month.

(b) The weight of a 1-inch diameter steel ball bearing in meganewtons.

(c) The mass of a 1 kg object on the surface of the moon.

(d) The equivalent rate of work in watts of 4 horsepower.

Solution:

(a) If hair grows 0.75 in. in one month, then

$$v = \frac{0.75 \text{ in.}}{1 \text{ month}} \frac{0.0254 \text{ m}}{1\text{in.}} \frac{1 \text{ month}}{30 \text{ days}} \frac{1 \text{ day}}{24 \text{ hours}} \frac{1 \text{ hour}}{60 \text{ min.}} \frac{1 \text{ min.}}{60 \text{ s}} = 7.350 \times 10^{-9} \text{ m/s}$$

(b) From the inside front cover, the density of steel is $\rho = 7840$ kg/m^3. Since the diameter is $d = 1$ in. $= 0.0254$ m, the volume of the 1-in. diameter sphere is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.0254 \text{ m})^3 = 6.864 \times 10^{-5} \text{ m}^3$$

So that its weight is

$$W = \rho g V = (7840 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(6.864 \times 10^{-5} \text{ m}^3) = 5.279 \text{ N} = 5.279 \times 10^{-6} \text{ MN}$$

(c) The mass of an object doesn't change, even if the weight of the object does. The mass remains 1 kg.

(d) From the inside front cover, 1 hp = 746 W. Therefore

$$4 \text{ hp} = (4 \text{ hp})\left(\frac{746 \text{ W}}{1 \text{ hp}}\right) = 2984 \text{ W}$$

1.13 The unit for dynamic viscosity in the SI system is newton-seconds per square meter, or pascal-seconds (N-s/m^2 = Pa-s). How can that unit be rewritten by using the basic relationships described by Newton's law for force and acceleration?

Notes: The only units needed are kg, m, and s.

Solution: By definition, a Newton is a kilogram-meter per square second. Therefore, the unit for dynamic viscosity can be written as:

$$\frac{\text{Ns}}{\text{m}^2} = \left(\frac{\text{kgm}}{\text{s}^2}\right)\frac{\text{s}}{\text{m}^2} = \frac{\text{kg}}{\text{m-s}}$$

1.14 The unit for dynamic viscosity in Problem 1.11 is newton-seconds per square meter (N-s/m^2) and the kinematic viscosity is defined as the dynamic viscosity divided by the fluid density. Find at least one unit for kinematic viscosity.

Notes: The only units needed are m and s.

Solution: The units for kinematic viscosity can be written as:

$$\eta_k = \frac{\eta}{\rho} = \frac{\left(\dfrac{\text{Ns}}{\text{m}^2}\right)}{\left(\dfrac{\text{kg}}{\text{m}^3}\right)} = \frac{\text{Nsm}}{\text{kg}} = \frac{\left(\dfrac{\text{kg m}}{\text{s}^2}\right)\text{sm}}{\text{kg}} = \frac{\text{m}^2}{\text{s}}$$

1.15 A square surface has sides 1 m long. The sides can be split into decimeters, centimeters, or millimeters, where 1 m = 10 dm, 1 dm = 10 cm, and 1 cm = 10 mm. How many millimeters, centimeters, and decimeters equal 1 m? Also, how many square millimeters, square centimeters , and square decimeters equal a square meter?

Notes: This is fairly straightforward; Table 1.3(b) is useful.

Solution: From Table 1.3(b), one meter equals 10 dm, 100 cm and 1000 mm. Therefore, a square meter is:

$$1 \text{ m}^2 = (1000 \text{ mm})^2 = 1 \text{ million}(\text{ mm})^2$$
$$1 \text{ m}^2 = (100 \text{ cm})^2 = 10{,}000(\text{ cm})^2$$
$$1 \text{ m}^2 = (10 \text{ dm})^2 = 100(\text{ dm})^2$$

1.16 A volume is 1 tera (mm^3) large. Calculate how long the sides of a cube must be to contain that volume.

Notes: From Table 1.3 (b), tera is 1×10^{12}.

Solution: A tera (mm^3) is 10^{12} (mm^3). Therefore, this volume is

$$10^{12}\ \text{mm}^3 \left(\frac{1\ \text{m}}{1000\ \text{mm}}\right)^3 = 1000\ \text{m}^3$$

A cube with this volume has sides with length of 10 m.

1.17 A ray of light travels at a speed of 300 000 km/s = 3×10^8 m/s. How far will it travel in 1 ps, 1 ns, and 1 μs?

Notes: An interesting assignment is to have students submit string of the required length for the picosecond and nanosecond cases.

Solution: The distance traveled in a picosecond is:

$$x = v\Delta t = (3 \times 10^8\ \text{m/s})(10^{-12}\ \text{s}) = 3 \times 10^{-4}\ \text{m} = 0.3\ \text{mm}$$

In one nanosecond, light travels:

$$x = v\Delta t = (3 \times 10^8\ \text{m/s})(10^{-12}\ \text{s}) = 0.3\ \text{m} = 300\ \text{mm}$$

In one microsecond, light travels:

$$x = v\Delta t = (3 \times 10^8\ \text{m/s})(10^{-6}\ \text{s}) = 300\ \text{m}$$

Section 1.9

1.18 Two smooth flat surfaces are separated by a 10-μm-thick lubricant film. The viscosity of the lubricant is 0.100 Pa-s. One surface has an area of 1 dm^2 and slides over the plane surface with a velocity of 1 km/hr. Determine the friction force due to shearing of the lubricant film. Assume the friction force is the viscosity times the surface area times the velocity of the moving surface and divided by the lubricant film thickness.

Notes: This problem is simple if consistent units are used.

Solution: Rewriting terms into consistent units, 10 μm = 10^{-5} m, $0.100\text{Pas} = 0.100\ \text{Ns/m}^2$, $1\ \text{dm}^2 = (0.1\ \text{m})^2 = 0.01\ \text{m}^2$, and

$$1\frac{\text{km}}{\text{hr}} = \frac{1000\ \text{m}}{3600\ \text{s}} = 0.2778\ \text{m/s}$$

Therefore the friction force is given by:

$$F = \frac{\eta_0 A u_b}{h} = \frac{\left(0.1\dfrac{\text{Ns}}{\text{m}^2}\right)(0.001\ \text{m}^2)(0.2778\ \text{m/s})}{10^{-5}\ \text{m}} = 27.78\ \text{N}$$

1.19 A firefighter sprays water on a house. The nozzle diameter is small relative to the hose diameter, so the force on the nozzle from the water is

$$F = v\frac{dm_a}{dt}$$

where v is the water velocity and dm_a/dt is the water mass flow per unit time. Calculate the force the firefighter needs to hold the nozzle if the water mass flow is 3 tons/hr and the water velocity is 100 km/hr.

Notes: A ton can be misperceived; in metric units, a ton is 1000kg, while in English units a short tin is 2000 lb and a long ton is 2240lb. Since the units in this problem are metric, the metric interpretation is used. This problem is not difficult as long as consistent units are used.

Solution: The velocity can be written as:

$$100\frac{\text{km}}{\text{hr}} = \frac{100,000 \text{ m}}{3600 \text{ s}} = 27.78 \text{ m/s}$$

The mass flow rate can be rewritten as:

$$\frac{dm_a}{dt} = \frac{3 \text{ tons}}{\text{hr}} = \frac{3000 \text{ kg}}{3600 \text{ s}} = 0.8333 \text{ kg/s}$$

Therefore, the force needed is

$$F = v\frac{dm_a}{dt} = (27.78 \text{ m/s})(0.8333 \text{ kg/s}) = 23.15 \text{ N}$$

1.20 The mass of a car is 1346 kg. The four passengers in the car weigh 65.55 kg, 75.23 kg, 88.66 kg, and 91.32 kg. It is raining and the additional weight due to the water on the car is 1.349 kg. Calculate the total weight of the car, including the weight of the passengers and the weight of the water, using four significant figures.

Solution: The weight is

$$W_{\text{tot}} = 1346 \text{ kg} + 65.55 \text{ kg} + 75.23 \text{ kg} + 88.66 \text{ kg} + 91.32 \text{ kg} + 1.349 \text{ kg} \pm 0.5 \text{ kg} = 1668.109 \text{ kg} \pm 0.5kg \text{ kg}$$

Therefore the total weight is 1668 kg.

1.21 During an acceleration test of a car the acceleration was measured to be 1.4363 m/s^2. Because slush and mud adhered to the bottom of the car, the weight was estimated to be 1400 $\pm$ 100 kg. Calculate the force driving the car and indicate the accuracy.

Solution: Since the force is given by $P = m_a a$, the force is given by

$$P = m_a a = (1400 \text{ kg} \pm 100 \text{ kg})(1.4363 \text{ m/s}^2 = 2010.82 \pm 143.6 \text{ N} = \begin{cases} 2154.45 \text{ N} \\ 1867.19 \text{ N} \end{cases}$$

The accuracy is only one digit or 2×10^3 N.

Chapter 2

Load, Stress and Strain

Qualitative Problems

2.16 Give three examples of (a) static loads; (b) sustained loads; (c) impact loads; and (d) cyclic loads.

There are many acceptable answers to this problem, some of which are:

(a) Examples of static loads include the force exerted by a hydraulic press to flatten a piece of metal, the preload in a tightened bolt, or the force in a tension test. Many sustained loads can also be taken as static loads.

(b) Sustained loads include weight loads of all kinds (such as the weight of a book on a bookshelf or a machine frame bearing against a foundation), wind loads on a windmill vane, the load on shoes when their user is standing, and the force exerted by hydraulic pumps when they are operated.

(c) Impact loads include the force exerted by a hammer on a nail, the force of a golf ball struck by a club, or a shopping cart colliding with an automobile door in a parking lot.

(d) Examples of cyclic loads include walking loads on artificial joints, bending stresses in rotating shafts, and surface stresses for a rolling element bearing.

2.17 Explain the sign convention for shear forces.

The sign convention for shear stresses is explained in Sec. 2.3. Basically, a shear stress is considered positive if it acts on a positive face (the face has an outward pointing normal that is in the positive direction of a right-handed coordinate system) and has a positive direction.

2.18 Explain the common sign conventions for bending moments. Which is used in this book?

The sign convention for bending is explained in Sec. 2.3 and Fig. 2.3. The two sign conventions are:

(a) A positive bending moment is defined as causing a positive stress for a positive displacement from the neutral axis according to a right-handed coordinate system. This is depicted in Fig. 2.3a.

(b) A positive bending moment is defined by it's vector, just as shear forces. That is, if a bending moment has its vector determined by a right-hand coordinate system, then a positively directed moment acting on a face with a positive outward pointing normal is positive. A negative moment acting on a negative face is also positive. This is shown in Fig. 2.3b.

The second of these sign conventions is generally used in this book.

2.19 Without the use of equations, explain a methodology for producing shear and moment diagrams.

First, one must be able to obtain the forces acting on a beam, including all reactions. Then sketch two axes that are associated with the beam. For the first, the shear diagram will be constructed. The shear diagram is the integral of the loads acting on the beam, calculated in the positive direction. Thus, starting at the left end of the beam and working right, calculate the integral of the load curve. This means that a step will occur on the shear diagram if a concentrated load is encountered, otherwise, the values can be obtained from the areas. Note that a positive force acting on the beam (i.e., one acting in a positive direction) causes a negative shear force according to the sign convention in Sec. 2.3. Once the shear diagram has been constructed, the bending moment diagram can be obtained using the same approach, but based on the shear diagram. As a check of calculations, the bending moment must equal zero at the right end of the beam.

2.20 Give two examples of scalars, vectors and tensors.

There are many scalars, such as the weight of an object, physical dimensions, surface roughness, color, pH of a solution, temperature, the number of runs in a baseball game, etc. Examples of vectors include position relative to another point, velocity, acceleration and the higher derivatives of position, and force. Examples of tensors are stress, strain and elastic modulus.

2.21 Explain the difference between plane stress and plain strain. Give an example of each.

Plane stress involves a condition where the stresses in one direction are zero. For example, if σ_z, τ_{xz} and τ_{yz} are all zero, then all of the stresses take place in one plane. Examples of plane stress are the surface or neutral axis of a beam (and the many applications for beams), contacting gear teeth, and thin-walled pressure vessels (as long as the radial stress can be neglected. Plane strain involves a situation where the strains in one direction vanish, such as ϵ_z, γ_{xz} and γ_{yz}. Examples include rolling of metal, orthogonal metal cutting, and strip drawing.

2.22 Without the use of equations, qualitatively determine the bending moment diagram for a bookshelf.

A bookshelf has a uniformly distributed load applied across the length. It generally has two supports that are symmetrically located an equal distance from each end, each of which takes one-half the total applied load. Considering the shear from one end to the other, the shear diagram is triangular until the support, then there is a downwards jump in the shear. The shear then increases until the second support, then takes another jump downwards. The shear continues to increase until the end of the beam. The moment is the integral of the shear diagram; parabolic profiles are present instead of linear ones.

2.23 Explain why $\tau_{xy} = \tau_{yx}$.

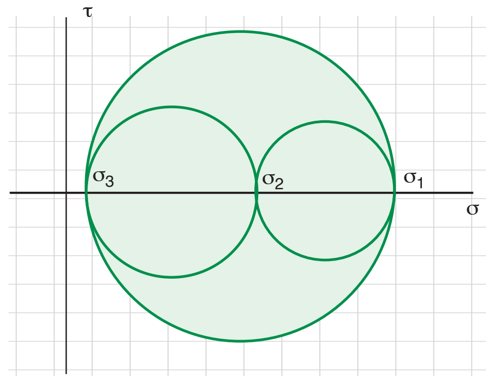
The basic reason is that moment equilibium must be preserved. For a three-dimensional stress state, the sum of the moments in the x-direction must equal zero. The components are associated with the product of the forces from these shear stresses and their moment arm. For a cubical stress element, the moment arms are equal.

2.24 Define and give two examples of (a) uniaxial stress state; (b) biaxial stress state and (c) triaxial stress state.

Uniaxial stress states are simple tension or compression loads. Examples include the leg of a chair, wire rope or chain supporting a chandelier, a wire hanger, a wire rope in a hoist, and a bumper made of a single material. Biaxial stress states occur in two directions; examples include torsion springs, pressure vessles (if the thickness stress is neglected) and gear teeth contacts. Most beams are also loaded biaxially. Examples of triaxial stress states include thick-walled pressure vessels and forging of workpieces with finite width and length.

2.25 Sketch and describe the characteristics of a three-dimensional Mohr's circle.

A generic three-dimensional Mohr's circle is shown below. The main characteristics are that the principal stresses are easily located as the intercepts with the abscissa, and the maximum shear stress is the radius of the largest circle.



2.26 What are the similarities and differences between deformation and strain?

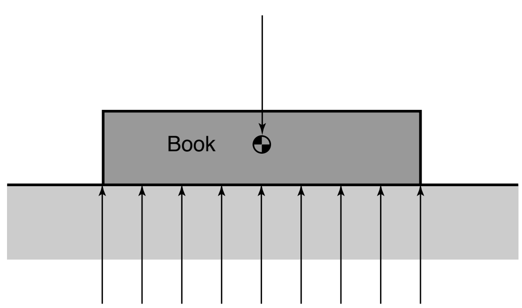
The main similarities are that they are associated with changes in dimensions of objects. Strain is deformation normalized by initial dimension. Another main difference is that strain thus is dimensionless, while deformation has units of length.

2.27 The text stated that 0°-45°-90° strain gage rosettes are common. Explain why.

Usually, when stress is to be measured on an object, an engineer has a good idea of the nature of the stress. Often, they know the direction of principal stresses, but not the magnitude. In such a circumstance, the 0°-45°-90° strain gage rosette is very useful, and will be aligned to coincide with principal stresses. Since Eq. (2.41) shows that the strains in the x-direction will be measured by the $0°$ strain gage, the stress in the y-direction will be measured by the $90°$ strain gage, and the $45°$ strain gage will measure a value if the expected principal direction varies from expectations. If the principal directions are not known, then Eq. (2.41) shows that ϵ_x will be measured by the $0°$ strain gage, ϵ_y will be measured by the $90°$ strain gage, and, since ϵ_x and ϵ_y are known, γ_{xy} can be obtained from the $45°$ strain gage.

2.28 Draw a free body diagram of a book on a table.

The main purpose is to demonstrate that the book results in a distributed load, or pressure, on the table, and not a concentrated force. There is a statically equivalent force that can be defined for determining reactions, but the book contacts the table over an area. An example of a free body diagram is shown below.



2.29 If the three principal stresses are determined to be 100 MPa, -50 MPa and 75 MPa, which is σ_2?

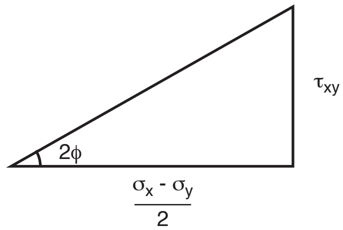
According to Eq. (2.18), $\sigma_1 \geq \sigma_2 \geq \sigma_3$, so that for these stresses, $\sigma_2 = 75$ MPa.

2.30 Derive Eq. (2.16).

Note from Eq. (2.15) that

$$\tan 2\phi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Consider the following right triangle:



From the Pythagorean theorem, the hypotenuse is

$$h = \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)}$$

Therefore,

$$\sin 2\phi = \frac{\tau_{xy}}{h} = \frac{\tau_{xy}}{\sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)}}$$

$$\cos 2\phi = \frac{(\sigma_x - \sigma_y)/2}{h} = \frac{(\sigma_x - \sigma_y)/2}{\sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)}}$$

Substitute these values into Eq. (2.13) to yield

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 \left(\frac{1}{h}\right) + \tau_{xy}^2 \left(\frac{1}{h}\right)$$

Factoring,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right] \left(\frac{1}{h}\right) = \frac{\sigma_x + \sigma_y}{2} + h^2 \left(\frac{1}{h}\right)$$

Substituting for h,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)}$$

The other principal stress can be obtained by using the related triangle with $-(\sigma_x - \sigma_y)/2$ and $-\tau_{xy}$ as the leg lengths.

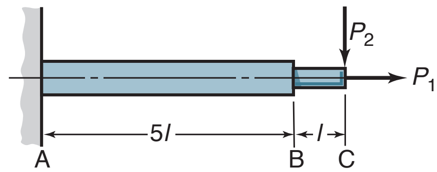
Quantitative Problems

2.31 The stepped shaft A-B-C shown in sketch a is loaded with the forces P_1 and/or P_2. Note that P_1 gives a tensile stress σ in B-C and $\sigma/4$ in A-B and that P_2 gives a bending stress σ at B and 1.5σ at A. What is the critical section

(a) If only P_1 is applied?

(b) If only P_2 is applied?

(c) If both P_1 and P_2 are applied?



Sketch a, used in Problems 2.31 and 2.32.

Solution:

(a) If only P_1 is applied, the critical section is the length B-C, since the tensile stress is highest in this section.

(b) If only P_2 is applied, the largest stress occurs at A.

(c) If both P_1 and P_2 are applied, the maximum stress in section A-B occurs at point A and is

$$\sigma_A = \frac{\sigma}{4} + 1.5\sigma = 1.75\sigma$$

The maximum stress in section B-C occurs at point B and is

$$\sigma_B = \sigma + \sigma = 2\sigma$$

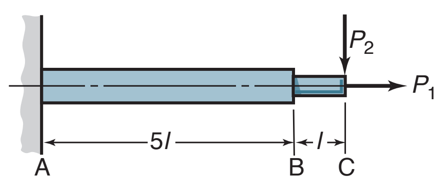
Therefore, the critical section is B-C, in particular, point B.

2.32 The stepped shaft in sketch a has loads P_1 and P_2. Find the load classification if P_1's variation is sinusoidal and P_2 is the load from a weight

(a) If only P_1 is applied

(b) If only P_2 is applied

(c) If both P_1 and P_2 are applied



Sketch a, used in Problems 2.31 and 2.32.

Solution:

(a) If only P_1 is applied, then the load is cyclic with respect to time, and is a normal load.

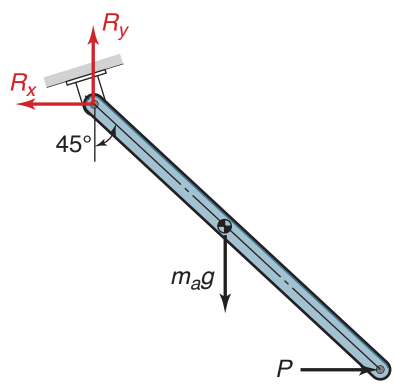
(b) If only P_2 is applied, the load is sustained with respect to time, and is both a shear load and a bending load. However, for long columns, bending is usually more important than shear, so it is reasonable to classify P_2 as a bending load.

(c) If both P_1 and P_2 are applied, then the load is cyclic with respect to time (although the mean stress is not zero), and the loading is combined.

2.33 A bar hangs freely from a frictionless hinge. A horizontal force P is applied at the bottom of the bar until it inclines 45° from the vertical direction. Calculate the horizontal and vertical components of the force on the hinge if the acceleration due to gravity is g, the bar has a constant cross section along its length, and the total mass is m_a. *Ans.* $R_x = \frac{1}{2}m_a g$, $R_y = m_a g$.

Solution:

The free body diagram of the bar is as shown below. Since the bar cross section is constant, we can put the center of gravity at the geometric center of the bar. The reactions at the hinge have been drawn in red. Notice the absence of a moment reaction, since the hinge was expressly described as frictionless (this is a common assumption for hinges).



Usually, it saves time to apply moment equilibrium before force equilibrium. This is true here as well. Therefore, summing moments about the hinge point, Eq. (2.2) gives:

$$\sum M = 0 = m_a g l \left(\frac{1}{2}\sin 45^\circ\right) - Pl\cos 45^\circ$$

or

$$P = \frac{m_a g}{2}$$

Taking equilibrium in the x-direction, Eq. (2.1) gives:

$$\sum P_x = 0 = -R_x + P$$

or

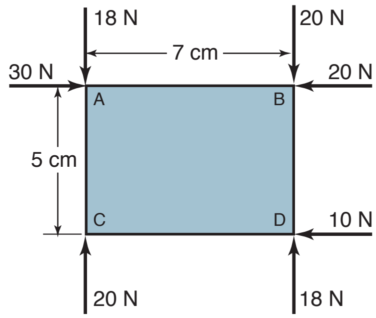
$$R_x = P = \frac{m_a g}{2}$$

Similarly, in the y-direction,

$$\sum P_y = 0 = R_y - m_a g$$

or $R_y = m_a g$.

2.34 Sketch b shows the forces acting on a rectangle. Is the rectangle in equilibrium? *Ans.* No.



Sketch b, used in Problem 2.34

Solution: Taking the sum of forces in the x-direction,

$$\sum F_x = 30\text{ N} - 20\text{ N} - 10\text{ N} = 0$$

Therefore, equilibrium in the x-direction is satisfied. Taking equilibrium in the y-direction gives

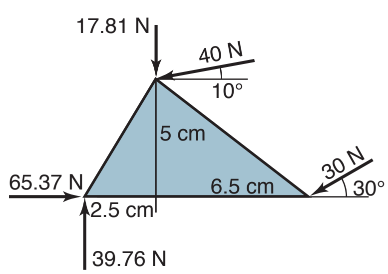
$$\sum F_y = -18\text{ N} + 20\text{ N} - 20\text{ N} + 18\text{ N} = 0$$

Therefore, equilibrium in the y-direction is satisfied. Taking moments about point A,

$$\sum M_A = (20\text{ N})(7\text{ cm}) + (10\text{ N})(5\text{ cm}) - (18\text{ N})(7\text{ cm}) = 64\text{ N-cm}$$

Therefore, moment equilibrium is not satisfied, and the rectangle is not in equilibrium.

2.35 Sketch c shows the forces acting on a triangle. Is the triangle in equilibrium? *Ans.* Yes.



Sketch c, used in Problem 2.35

Solution: Taking the sum of forces in the x-direction,

$$\sum F_x = 65.37\text{ N} - 40\text{ N}(\cos 10°) - 30\text{ N}(\cos 30°) = 0.00$$

Therefore, equilibrium in the x-direction is satisfied. Taking equilibrium in the y-direction gives

$$\sum F_y = 39.76\text{ N} - 17.81\text{ N} - 40\text{ N}(\sin 10°) - 30\text{ N}(\sin 30°) = 0.00$$

Therefore, equilibrium in the y-direction is satisfied. Taking moments about point A,

$$\sum M_A = (39.76\text{ N})(2.5\text{ cm}) - (40\text{ N})(\cos 10°)(5\text{ cm}) + (30\text{ N})(\sin 30°)(6.5\text{ cm}) = 0.00$$

Therefore, the triangle is in equilibrium.