Chapter 1

1.1

$$f(t) = 5 \sin(10t - 2.5) \qquad (1)$$

Eq. (1) can be rewritten as

$$f(t) = 5 \left[\sin \omega t \cos 2.5 - \cos \omega t \cdot \sin 2.5\right]$$

$$= c_1 \sin \omega t + c_2 \cos \omega t \qquad (2)$$

where $c_1 = 5 \cos \underset{143.2391^\circ}{\underset{\shortparallel}{2.5}} = 5(-0.8011) = -4.0057$

and $c_2 = -5 \sin 2.5 = -5(0.5985) = -2.9925$

$f(t)$ is expressed as a sum of sine and cosine functions with $c_1 = -4.0057$ and $c_2 = -2.9925$.

1.2

(a)

$$x_1(t) = 5 \sin 20t \quad , \quad x_2(t) = 8 \cos\left(20t + \frac{\pi}{3}\right) \qquad (1)$$

$$x(t) = x_1(t) + x_2(t) = 5 \sin 20t + 8 \cos\left(20t + \frac{\pi}{3}\right) \qquad (2)$$

$x(t)$ as a cosine function with a phase angle.

Eq. (2) can be rewritten as

$$x(t) = 5 \sin 20t + 8\left(\cos 20t \cdot \cos \frac{\pi}{3} - \sin 20t \cdot \sin \frac{\pi}{3}\right)$$

$$= \cos 20t \left(8 \cos \frac{\pi}{3}\right) - \sin 20t \left(-5 + 8 \sin \frac{\pi}{3}\right) \qquad (3)$$

or

$$x(t) = A \cos(20t + \phi)$$

$$= A \cos 20t \cdot \cos\phi - A \sin 20t \cdot \sin\phi$$

By equating Eqs. (3) and (4), we obtain

$$A \cos\phi = 8 \cos\frac{\pi}{3} = 8(0.5) = 4.0$$

$$A \sin\phi = -5 + 8 \sin\frac{\pi}{3} = -5 + 8(0.8660) = 1.9282$$

Thus $A = \sqrt{(A\cos\phi)^2 + (A \sin\phi)^2} = \sqrt{4.0^2 + 1.9282^2}$

$$= \sqrt{16.0 + 3.7179} = \sqrt{19.7179} = 4.4405$$

and $\phi = \tan^{-1}\left(\frac{A \sin\phi}{A\cos\phi}\right) = \tan^{-1}\left(\frac{1.9282}{4.0}\right) = \tan^{-1}(0.48205)$

$$= 25.7364^\circ \text{ or } 0.4492 \text{ rad}$$

$$\therefore x(t) = 4.4405 \cos(20t + 0.4492)$$

$$= 4.4405 \cos(20t + 25.7364^\circ) \qquad (5)$$

Problem 1.2 p.1/2

(b) $x(t)$ as a sine function with a phase angle.

Eq. (2) can be rewritten as

$$x(t) = 5 \sin 20t + 8 \left(\cos 20t \cdot \cos \frac{\pi}{3} - \sin 20t \cdot \sin \frac{\pi}{3}\right)$$

$$= \sin 20t \left(5 - 8 \sin \frac{\pi}{3}\right) + \cos 20t \left(8 \cos \frac{\pi}{3}\right) \qquad (6)$$

or

$$x(t) = A \cdot \sin 20t \cdot \cos\phi + A \cos 20t \cdot \sin\phi \qquad (7)$$

We obtain, by comparing Eqs. (6) and (7),

$$A \cos\phi = 5 - 8 \sin \frac{\pi}{3} = -1.9282 \qquad (8)$$

$$A \sin\phi = 8 \cos \frac{\pi}{3} = 4.0 \qquad (9)$$

so that

$$A = \sqrt{(A\cos\phi)^2 + (A \sin\phi)^2}$$

$$= \sqrt{(-1.9282)^2 + (4.0)^2} = 4.4405$$

and

$$\phi = \tan^{-1}\left(\frac{A \sin\phi}{A \cos\phi}\right) = \tan^{-1}\left(\frac{4.0}{-1.9282}\right) = \tan^{-1}(-2.0745)$$

$$= -64.2636^\circ = -1.1216 \text{ rad}$$

or $180^\circ - 64.2636^\circ$

$= 115.7372^\circ$

Since cosine is negative and sine is positive,

$$\phi = 115.7372^\circ \text{ or } 2.0200 \text{ rad}.$$

$$\therefore x(t) = 4.4405 \sin(20t + 2.0200)$$

$$\text{or} \quad 4.4405 \sin(20t + 115.7372^\circ)$$

Problem 1.2 p.2/2

1.3

$$x_1(t) = 6 \sin 30t$$

$$x_2(t) = 4 \cos\left(30t + \frac{\pi}{4}\right)$$

$$x(t) = x_1(t) - x_2(t) = 6 \sin 30t - 4 \cos\left(30t + \frac{\pi}{4}\right) \quad (1)$$

(a) $x(t)$ as a sine function with a phase angle.

Eq. (1) can be rewritten as

$$x(t) = 6 \sin 30t - 4\left(\cos 30t \cdot \cos\frac{\pi}{4} - \sin 30t \cdot \sin\frac{\pi}{4}\right) \quad (2)$$

$$= \sin 30t\left(6 + 4 \sin\frac{\pi}{4}\right) + \cos 30t\left(-4 \cos\frac{\pi}{4}\right) \quad (3)$$

By expressing Eq. (3) as

$$x(t) = A \cdot \sin(30t + \phi)$$

$$= A \sin 30t \cdot \cos\phi + A \cos 30t \cdot \sin\phi \quad (4)$$

Eqs. (3) and (4) yield:

$$A \cos\phi = 6 + 4 \sin\frac{\pi}{4} = 6 + 4(0.7071) = 8.8284$$

$$A \sin\phi = -4 \cos\frac{\pi}{4} = -4(0.7071) = -2.8284$$

where

$$A = \sqrt{(A\cos\phi)^2 + (A\sin\phi)^2} = \sqrt{(8.8284)^2 + (-2.8284)^2}$$

$$= \sqrt{77.9406 + 7.9998} = \sqrt{85.9404} = 9.2704$$

and $\phi = \tan^{-1}\left(\frac{A \cdot \sin\phi}{A \cos\phi}\right) = \tan^{-1}\left(-\frac{2.8284}{8.8284}\right) = \tan^{-1}(-0.3204)$

$$= -17.7642^\circ = -0.3100 \text{ rad}$$

$$\therefore x(t) = 9.2704 \sin(30t - 0.31) = 9.2704 \sin(30t - 17.7642^\circ) \quad (5)$$

Problem 1.3 p.1/2

(b) $x(t)$ as a cosine function with a phase angle:

Eq. (2) can be rewritten as

$$x(t) = \cos 30t \left(-4\cos\frac{\pi}{4}\right) - \sin 30t \left(-6 - 4\sin\frac{\pi}{4}\right) \quad (6)$$

By expressing Eq. (6) as

$$x(t) = A\cos(30t - \phi) = A\cos 30t \cdot \cos\phi - A\sin 30t \cdot \sin\phi \quad (7)$$

By equating Eqs. (6) and (7), we find

$$A\cos\phi = -4\cos\frac{\pi}{4} = -2.8284 \quad (8)$$

$$A\sin\phi = -6 - 4\sin\frac{\pi}{4} = -8.8284 \quad (9)$$

So that

$$A = \sqrt{(A\cos\phi)^2 + (A\sin\phi)^2} = \sqrt{(-2.8284)^2 + (-8.8284)^2}$$

$$= \sqrt{7.9998 + 77.9406} = \sqrt{85.9404} = 9.2704$$

and

$$\phi = \tan^{-1}\left(\frac{A\sin\phi}{A\cos\phi}\right) = \tan^{-1}\left(\frac{-8.8284}{-2.8284}\right) = \tan^{-1}(3.1213)$$

$$= \tan^{-1}(178.8369^\circ) = 89.6796^\circ \text{ or } 180^\circ + 89.6796^\circ$$

$$= 1.5652 \text{ rad or } 4.7068 \text{ rad}$$

Since both $\sin\phi$ and $\cos\phi$ are negative, the correct value of ϕ is 4.7068 rad or 269.6796°

$$\therefore \quad x(t) = 9.2704\cos(30t - 4.7068)$$

$$= 9.2704\cos(30t - 269.6796^\circ) \quad (10)$$

Problem 1.3 p.2/2

1.4

$$x_1(t) = 5 \cos \omega t, \quad x_2(t) = 10 \cos(\omega t + 1) \qquad (1)$$

(a) Using trigonometric relations:

$$x(t) = x_1(t) + x_2(t) = 5 \cos \omega t + 10 \cos(\omega t + 1) \qquad (2)$$

$$= 5 \cos \omega t + 10 (\cos \omega t \cdot \cos 1 - \sin \omega t \cdot \sin 1)$$

$$= \cos \omega t (5 + 10 \cos 1) - \sin \omega t (10 \sin 1) \qquad (3)$$

By expressing Eq. (3) as

$$x(t) = A \cos(\omega t + \phi) = A(\cos \omega t \cdot \cos \phi - \sin \omega t \cdot \sin \phi)$$

$$= \cos \omega t (A \cos \phi) - \sin \omega t (A \sin \phi) \qquad (4)$$

we obtain, by comparing Eqs. (3) and (4),

$$A \cos \phi = 5 + 10 \cos 1$$

$$A \sin \phi = + 10 \sin 1$$

$$\therefore A = \sqrt{(A \cos \phi)^2 + (A \sin \phi)^2} = \sqrt{(5 + 10 \cos 1)^2 + (10 \sin 1)^2}$$

$$= \sqrt{[5 + 10 * 0.5403]^2 + [10 * 0.8417]^2}$$

$$= (108.2224 + 70.8459)^{\frac{1}{2}} = 13.3816$$

and

$$\phi = \tan^{-1}\left(\frac{10 \sin 1}{5 + 10 \cos 1}\right) = \tan^{-1}\left(\frac{8.417}{10.403}\right)$$

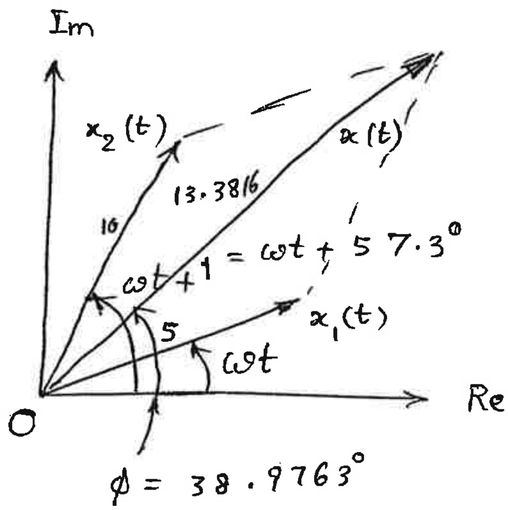
$$= \tan^{-1}(0.8091) = 0.6803 \text{ rad} = 38.9763^\circ$$

$$\therefore x(t) = x_1(t) + x_2(t) = 13.3816 \cos(\omega t + 0.6803)$$

Problem 1.4 p.1/2

(b) Using vectors:

For an arbitrary value of ωt, the harmonic motions $x_1(t)$ and $x_2(t)$ can be denoted graphically as shown in the following figure.



(c) Using complex number representation

Given $x_1(t)$ and $x_2(t)$ can be expressed as complex numbers:

$$x_1(t) = Re\left[A_1 e^{i\omega t}\right] \equiv Re\left[5\, e^{i\omega t}\right]$$

$$x_2(t) = Re\left[A_2 e^{i(\omega t+1)}\right] \equiv Re\left[10\, e^{i(\omega t+1)}\right]$$

where $A_1 = \sqrt{(5 + 10\cos 1)^2 + 0} = 10.403$

and $A_2 = \sqrt{(0 + 10 \sin 1)^2} = 8.417$

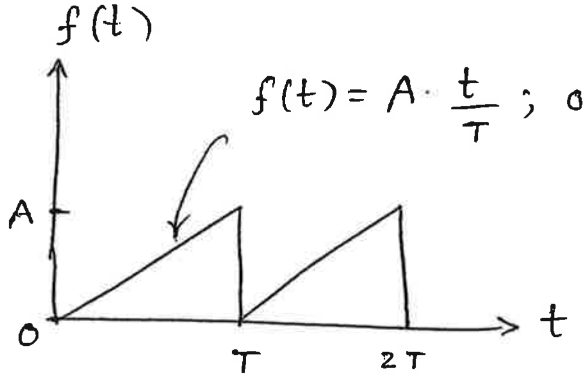
Thus $x(t) = Re\left[A\, e^{i(\omega t + \phi)}\right] = x_1 + x_2$

with $A_1 = \sqrt{A_1^2 + A_2^2} = 13.3816$

and $\phi = \tan^{-1}\left(\frac{A_2}{A_1}\right) = 38.9763^0$ or 0.6803 rad

Problem 1.4 p.2/2

1.6



$$f(t) = A \cdot \frac{t}{T}\ ; \quad 0 \le t \le T$$

$$\left\{ \begin{array}{l} T = \dfrac{2\pi}{\omega_0} \\ \omega_0 = \dfrac{2\pi}{T} \end{array} \right.$$

$$a_n = \frac{2}{T}\int_0^T A \cdot \frac{t}{T} \cos\left(\overbrace{\frac{2\pi n t}{T}}^{= n\omega_0 t}\right) \cdot dt$$

$$= \frac{2A}{T^2} \cdot \int_0^t t \cdot \cos\frac{2\pi n \cdot t}{T}\, dt$$

$$= \frac{2A}{T^2}\left[\frac{T^2}{(2\pi n)^2} \cdot \cos\frac{2\pi n}{T} \cdot t + \frac{t \cdot T}{2\pi n} \cdot \sin\frac{2\pi n}{T} \cdot t\right]_0^T$$

$$= \frac{2A}{\cancel{T^2}} \cdot \left[\frac{\cancel{T^2}}{4\pi^2 n^2} \cdot \left(\cos\frac{2\pi n \cdot T}{T} - 1\right) + \frac{\cancel{T^2}}{2\pi n} \cdot \left(\sin\frac{2\pi n \cdot T}{T}\right)\right]$$

$$= \frac{A}{2\pi^2 n^2}\left(\cos 2\pi n - 1\right) + \frac{A}{\pi n}\left(\underbrace{\sin 2\pi n}_{= 0}\right)$$

$$b_n = \frac{2}{T}\int_0^T A \cdot \frac{t}{T} \sin\frac{2\pi n t}{T} \cdot dt$$

$a_0 = A\,, \quad a_n = 0\ ; n = 1, 2, 3, \cdots\ ; \quad b_n = -\dfrac{A}{n\pi}\ ; \ n = 1, 2, 3, \ldots$

$$\therefore \ f(t) = A\left(\frac{1}{2} - \frac{1}{\pi}\sum_{n=1}^{\infty} \frac{1}{n} \sin\frac{2\pi n t}{T}\right)$$

1.7

$$x(t) = \begin{cases} A \sin \frac{2\pi t}{\tau} & , \quad 0 \le t \le \frac{\tau}{2} \\ 0 & , \quad \frac{\tau}{2} \le t \le \tau \end{cases}$$

$$a_0 = \frac{2}{\tau} \int_0^{\tau} x(t)\, dt = \frac{2A}{\tau} \int_0^{\tau/2} \sin \frac{2\pi t}{\tau}\, dt = \frac{2A}{\tau} \left(-\frac{\tau}{2\pi} \cdot \cos \frac{2\pi t}{\tau} \right)_0^{\tau/2}$$

$$= \frac{2A}{\pi}$$

$$a_n = \frac{2}{\tau} \int_0^{\tau} x(t) \cos n\omega t\, dt = \frac{2A}{\tau} \int_0^{\tau/2} \sin \frac{2\pi t}{\tau} \cdot \cos n\omega t \cdot dt \qquad \text{---- } (E_1)$$

Using the relation $\sin m\omega t \cdot \cos n\omega t = \frac{\sin (m+n)\omega t + \sin (m-n)\omega t}{2}$, Eq. (E_1) can be rewritten as

$$a_n = \frac{A}{\tau} \int_0^{\pi/\omega} \left[\sin (1+n)\omega t + \sin (1-n)\omega t \right] dt$$

when $n = 1$, $a_1 = \frac{A}{\tau} \int_0^{\pi/\omega} \sin 2\omega t \cdot dt = 0$

when $n = 2, 3, 4, \ldots$, $a_n = \frac{A}{\tau} \left[-\frac{\cos (1+n)\omega t}{(1+n)\omega} - \frac{\cos (1-n)\omega t}{(1-n)\omega} \right]_0^{\pi/\omega}$

$$= \frac{A}{2\pi} \left[\frac{1 - \cos (1+n)\pi}{1+n} + \frac{1 - \cos (1-n)\pi}{1-n} \right]$$

$$= \begin{cases} 0 & \text{if } n \text{ is odd} \\ -\frac{2A}{(n-1)(n+1)\pi} & \text{if } n \text{ is even} \end{cases}$$

Similarly

$$b_n = \frac{2}{\tau} \int_0^{\tau} x(t) \sin n\omega t\, dt = \frac{2A}{\tau} \int_0^{\tau/2} \sin \frac{2\pi t}{\tau} \cos n\omega t\, dt$$

$$= \frac{A}{\tau} \int_0^{\tau/2} \left[\cos (1-n)\omega t - \cos (1+n)\omega t \right] dt$$

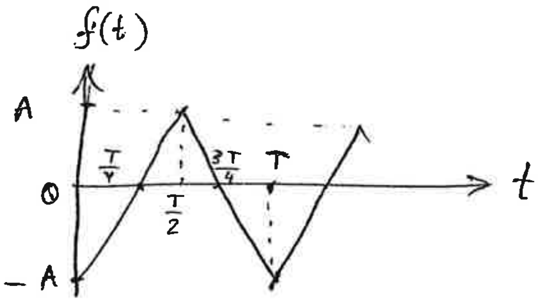
when $n = 1$, $b_1 = \frac{A}{\tau} \int_0^{\pi/\omega} (dt - \cos 2\omega t)\, dt = \frac{A}{2}$

when $n = 2, 3, 4, \ldots$, $b_n = \frac{A}{\tau} \left[\frac{\sin (1-n)\omega t}{(1-n)\omega} - \frac{\sin (1+n)\omega t}{(1+n)\omega} \right]_0^{\pi/\omega} = 0$

$$\therefore\ x(t) = \frac{A}{\pi} + \frac{A}{2} \sin \omega t - \frac{2A}{\pi} \sum_{n=2,4,6,\ldots}^{\infty} \frac{\cos n\omega t}{(n^2 - 1)}$$

1.9

$$f(t) = \begin{cases} \left(\frac{4A}{T}\right)t - A \;;\; 0 \le t \le \frac{T}{2} \\ A - \frac{4A}{T}\left(t - \frac{T}{2}\right) \\ \quad = 3A - \frac{4At}{T} \;;\; \frac{T}{2} \le t \le T \end{cases}$$



$$a_0 = \frac{2}{T}\int_0^T F(t)\cdot dt = \frac{2}{T}\left[\int_0^{T/2}\left\{\left(\frac{4A}{T}\right)t - A\right\}dt + \int_{T/2}^{T}\left\{A - \frac{4A}{T}\left(t - \frac{T}{2}\right)\right\}dt\right.$$

$$= \frac{2}{T}\left[\frac{4A}{T}\cdot\left(\frac{t^2}{2}\Big|_0^{T/2}\right) - A\cdot\left(t\right)_0^{T/2} + A\cdot\left(t\right)_{T/2}^{T} - \frac{4A}{T}\cdot\left(\frac{t^2}{2}\right)_{T/2}^{T} + \overset{2A}{\left(\frac{4AT}{2T}\right)}\cdot\left(t\right)_{T/2}^{T}\right]$$

$$= \frac{2}{T}\left[\frac{4A}{2T}\cdot\left(\frac{T^2}{4}\right) - A\cdot\frac{T}{2} + A\cdot\left(T - \frac{T}{2}\right) - \frac{4A}{2T}\left(T^2 - \frac{T^2}{4}\right) + 2A\left(T - \frac{T}{2}\right)\right]$$

$$= \frac{2}{T}\left[\frac{AT}{2} - \cancel{\frac{AT}{2}} + \cancel{\frac{AT}{2}} - \frac{2A}{T}\left(\frac{3T^2}{4}\right) + \cancel{2}A\cdot\frac{T}{\cancel{2}}\right]$$

$$= \frac{2}{T}\left[\frac{AT}{2} - \frac{3AT}{2} + \frac{2}{2}AT\right] = 0$$

Problem 1.9 p.1/3

$$a_n = \frac{2}{T}\int_0^{T/2}\left(\frac{4A}{T}\cdot t - A\right)\cos n\omega_0 t \; dt$$

$$+ \frac{2}{T}\int_{T/2}^{T}\left[3A - \frac{4A}{T}\cdot t\right] \cdot \cos n\omega_0 t \cdot dt$$

$$= \frac{2}{T}\cdot\frac{4A}{T}\cdot\int_0^{T/2} t\cdot\cos n\omega_0 t\; dt - \frac{2A}{T}\cdot\int_0^{T/2}\cos n\omega_0 t\; dt$$

$$+ \frac{2}{T}(3A)\cdot\int_{T/2}^{T}\cos n\omega_0 t \cdot dt$$

$$-\underbrace{\left(\frac{2}{T}\cdot\frac{4A}{T}\right)}_{=\left(\frac{8A}{T^2}\right)}\cdot\int_{T/2}^{T} t\cdot\cos n\omega_0 t\cdot dt$$

$$= \frac{8A}{T^2}\left[\frac{1}{n^2\omega_0^2}\cos n\omega_0 t + \frac{t}{n\omega_0}\sin n\omega_0 t\right]_0^{T/2}$$

$$-\frac{2A}{T}\left[\frac{1}{n\omega_0}\cdot\sin n\omega_0 t\right]_0^{\frac{T}{2}} + \frac{6A}{T}\left[\frac{1}{n\omega_0}\sin n\omega_0 t\right]_0^{T/2}$$

$$-\frac{8A}{T^2}\left[\frac{1}{n^2\omega_0^2}\cos n\omega_0 t + \frac{t}{n\omega_0}\sin n\omega_0 t\right]_{T/2}^{T} \qquad (A)$$

where $\int \cos ax \; dx = \frac{1}{a}\sin ax$

and $\int_x x\cos ax\; dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax$

with $a = n\omega_0$ were used.

Problem 1.9 p. 2/3

Eq. (A) gives

$$\frac{8A}{T^2}\cdot\frac{1}{n^2\omega_0^2}\left[\cos\frac{n\omega_0 T}{2}\right]$$

$$+\frac{8A}{T^2}\cdot\frac{T}{2n\omega_0}\cdot\left[\sin\frac{n\omega_0 T}{2}\right]-\frac{2A}{Tn\omega_0}\cdot\sin\frac{n\omega_0 T}{2}+\frac{6A}{Tn\omega_0}\cdot\sin\frac{n\omega_0 T}{2}$$

$$-\frac{8A}{T^2}\cdot\frac{1}{n^2\omega_0^2}\cos n\omega_0 T-\frac{8A}{Tn\omega_0}\cdot\sin n\omega_0 T$$

$$+\frac{8A}{T^2\cdot n^2\omega_0^2}\cos\frac{n\omega_0 T}{2}+\frac{4A}{Tn\omega_0}\sin\frac{n\omega_0 T}{2}$$

$$=\frac{16A}{T^2\cdot n^2\omega_0^2}\cdot\cos\frac{n\omega_0 T}{2}+\sin\frac{n\omega_0 T}{2}\left[\frac{4A}{Tn\omega_0}-\frac{2A}{Tn\omega_0}+\frac{6A}{Tn\omega_0}+\frac{4A}{Tn\omega_0}\right]$$

$$-\frac{8A}{T^2n^2\omega_0^2}\cos n\omega_0 T-\frac{8A}{Tn\omega_0}\sin n\omega_0 T$$

$$=\frac{4A}{n^2\pi^2}\cos n\pi+\frac{6A}{n\pi}\sin n\pi$$

$$-\frac{2A}{n^2\pi^2}\cos 2n\pi-\frac{4A}{n\pi}\sin 2n\pi$$

If $n=$ even $=2, 4, 6, \ldots$; $\cos n\pi = 1$

$\cos 2\pi n = 1$

Problem 1.9 p.3/3

1.11

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt \qquad (1)$$

since $f(t) = 0$ for $t < 0$, Eq. (1) becomes

$$F(\omega) = \int_{0}^{\infty} f(t)\, e^{-i\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-t}\, e^{-i\omega t}\, dt = \int_{0}^{\infty} e^{-(1+i\omega)t}\, dt$$

$$= \frac{1}{-(1+i\omega)}\, e^{-(1+i\omega)t} \Big|_{0}^{\infty}$$

$$= \frac{1}{1+i\omega}$$

1.12

$$F_s(\omega) = \int_{0}^{\infty} f(t) \cdot \sin \omega t\, dt \;;\; \omega > 0$$

$$= \int_{0}^{\infty} e^{-at} \sin \omega t\, dt \;;\; a > 0$$

$$= \frac{e^{-at}\{-a \sin \omega t - \omega \cos \omega t\}}{a^2 + \omega^2}\Bigg]_{t=0}^{\infty}$$

$$= 0 - \frac{\{e^{-0}\,[0 - \omega]\}}{a^2 + \omega^2}$$

$$= \frac{\omega}{a^2 + \omega^2}$$

1.13

$$F_s(\omega) = \int_0^\infty f(t) \cdot \sin \omega t \cdot dt \quad ; \quad \omega > 0$$

$$= \int_0^\infty t\, e^{-t} \sin \omega t \cdot dt \quad ; \quad t \geq 0 \qquad (1)$$

From Tables of Integrals, Eq. (1) can be expressed as

$$F_s(\omega) = \left\{ \frac{t\, e^{-t}}{a^2+\omega^2} (a \sin \omega t - \omega \cos \omega t) - \frac{e^{-t}}{(1+\omega^2)^2} \left[(1-\omega^2) \sin \omega t + 2\omega \cos \omega t \right] \right\}_{t=0}^{\infty}$$

with $a = -1$.

$$= \left\{ \frac{\infty \cdot e^{-\infty}}{1+\omega^2} \left(-\sin \omega(\infty) - \omega \cos \omega(\infty) \right) - \frac{e^{-\infty}}{(1+\omega^2)^2} \left[(1-\omega^2) \sin \omega t + 2\omega \cos \omega t \right] - 0 + \frac{e^{-0}}{(1+\omega^2)^2} \left[0 + 2\omega \right] \right\}$$

$$= \frac{2\omega}{(1+\omega^2)^2}$$

1.14

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-i\omega t}\, dt \qquad (1)$$

For $f(t) = 0$ when $t < 0$, Eq. (1) becomes

$$F(\omega) = \int_{0}^{\infty} f(t)\, e^{-i\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-at}\, e^{-i\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+i\omega)t}\, dt$$

$$= \frac{1}{-(a+i\omega)}\, e^{-(a+i\omega)t} \Big|_{t=0}^{\infty}$$

$$= \frac{1}{(a+i\omega)}$$