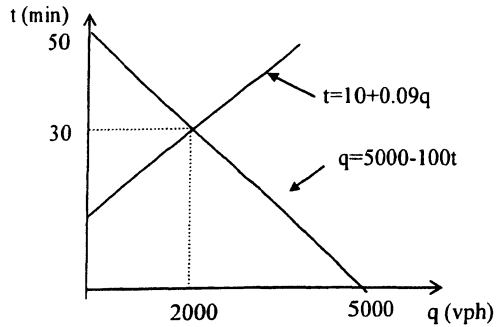


**CHAPTER 2
TRANSPORTATION ECONOMICS**

<2.1>

$$t = 10 + 0.09q$$

$$q = 5000 - 100t$$



a)

$$t = 10 + 0.01q \Rightarrow q = \frac{t - 10}{0.01} = 100t - 1000$$

$$100t^* - 1000 = 5000 - 100t^*$$

$$200t^* = 6000$$

$$t^* = 30 \text{ min}$$

$$q^* = 5000 - 100 \times 30 = 2000 \text{ veh / hr}$$

b) $L = 22.5$ miles, where $L =$ length in miles.

$$t = 30 \text{ min}$$

$$v = \frac{22.5}{30} \times 60 = 45 \text{ mph}$$

c)

$$t^* \cdot 10 + 0.005q \Rightarrow q^* = 200t - 2000$$

$$200t^* - 2000 = 5000 - 100t^*$$

$$300t^* = 7000$$

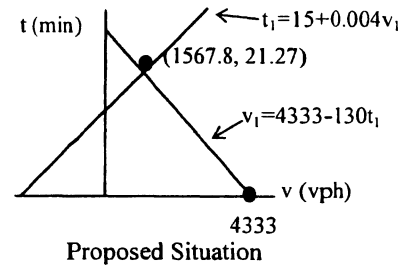
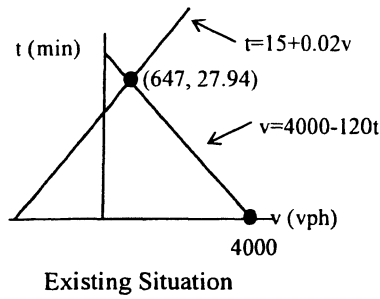
$$t^* = 23.33 \text{ min}$$

$$q^* = (5000 - 100) \times 23.33 = 2667 \text{ veh / hr}$$

$$v = \frac{22.5}{23.3} \times 60 = 58 \text{ mph}$$

<2.2>

a)



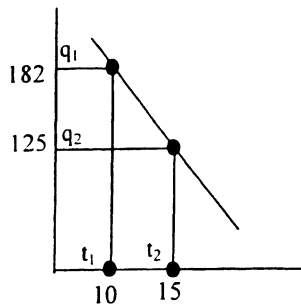
b) $v = 4333 - 130(15 + 0.004v)$
 $v = 4333 - 1950 - 0.52v$
 $1.52v = 2383$

$v = 1567.8 \text{ veh / hr.}$
 $t = 15 + (0.004 \times 1567.8) = 21.27 \text{ min.}$

c) $l = 20 \text{ miles}$
 $t = 21.27 \text{ min}$
 $v = \frac{l}{t} \cdot 60 = \frac{20}{21.27} \times 60 = 56.42 \text{ mph}$

<2.3>

Assume that the demand function is valid in the time range between $t = 10$ & $t = 15$ min



$$\begin{aligned} \Delta \text{veh} - \text{min } s &= q_1 t_1 - q_2 t_2 \\ &= 182(10) - 125(15) \\ &= (125)(15) - (182)(10) \\ &= 55 \text{ veh} - \text{mins are lost due to congestion} \end{aligned}$$

<2.4>

Assuming the demand function to be linear we get the following equations:

$$2000 = \alpha - 1.5 \beta$$

$$1000 = \alpha - 2.0 \beta$$

Hence $\alpha = 5000$, and $\beta = 2000$ $q = 5000 - 2000 p$

a) when the fare is 50c, $q = 5000 - (2000 \times 0.5) = 4000$

b) If the transit system were free, $p = 0$, and $q = 5000$, which is the latent demand.

<2.5>

$$p_0 = 50¢ / \text{ride}$$

$$q_0 = 500,000 \text{ per / day}$$

$$p_1 = 60¢ / \text{ride}$$

$$q_1 = 470,000 \text{ per / day}$$

$$a) \quad \epsilon = \frac{(Q_1 - Q_0)(p_1 + p_2)/2}{(p_1 - p_0)(Q_1 + Q_2)/2} = \frac{(470 - 500)(50 + 60)/2}{(60 - 50)(500 + 470)/2} = -0.34 \text{ (inelastic)}$$

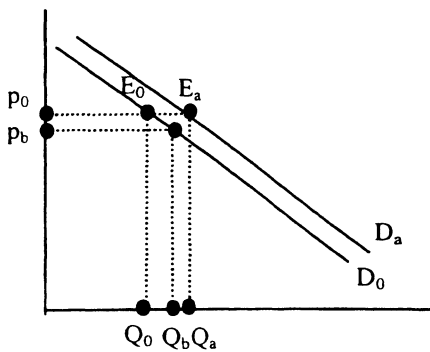
$$b) \quad 0.50 \times 500,000 = \$250,000 / \text{day}$$

$$0.60 \times 470,000 = \$282,000 / \text{day}$$

Total gain	= \$32,000 / day
------------	------------------

<2.6>

Assume (1) that the demand function remains unchanged, and (2) that the cost of improving the service is ignored.



a) Initial demand function (D_0)

$$Q_0 = 2125 - 1000 P_0$$

Option (a), demand function (D_a)

$$Q_a = 2150 - 1000 P_0$$

Additional Revenue ΔTR_a

$$= P_0 Q_a - P_0 Q_0$$

$$= 1.30 [(2150 - 1000 P_0) - (2125 -$$

$$1000 P_0)]$$

$$= \$32.50$$

b) New price reduced to $P_b = \$1.00/\text{ride}$

$$\Delta TR_b = 1.00 [2125 - 1000 (1.00)] - 1.30 [2125 - 1000 (1.30)]$$

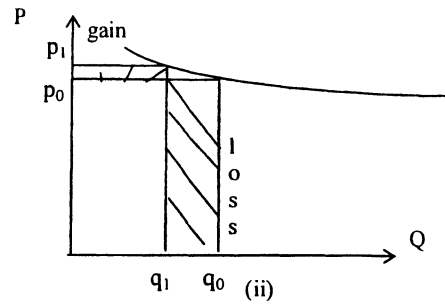
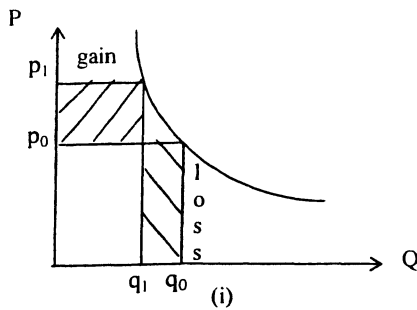
$$= \$52.50$$

Since $\Delta TR_b > \Delta TR_a$, option b is better.

<2.7>

(i) Price elasticity = - 0.4 (inelastic) ∴ price rise would lead to a fall in patronage but a total revenue gain. So fears are not justified.

(ii) In this case, $\epsilon_p = -1.3$ (elastic) in which case a rise in fare not only leads to a total loss of revenue, but a decrease in patronage. It would therefore not be advisable to raise the fare.



<2.8>

Using arc-price elasticity

$P_1 = \$100$; $P_2 = \$120$; $Q_1 = 5000$; and $Q_2 = ?$. Also $e = -1.2$

Notice that the price is elastic, and in general when price is elastic (-1.2), raising the price will result in total loss, but lowering the price will result in total gain.

$$-1.2 = \left(\frac{5000 - Q_2}{100 - 120} \right) \left(\frac{100 + 120}{5000 + Q_2} \right) \quad \text{therefore, } Q_2 = 4016$$

$$\begin{aligned} \text{Total difference in revenue} &= (100 \times 5000) - (120 \times 4016) \\ \text{Total loss} &= \$ -18,080 \end{aligned}$$

This loss should come as no surprise to the airlines and should have been taken into account before raising the fare.

<2.9>

This problem could be solved in several ways. The simplest is to follow classic economic style which states that price elasticity is the % change in quantity demanded which accompanies a 1% change in price.

$$\therefore e_p = \frac{10\%}{40\%} = 0.25 \text{ (inelastic)}$$

If scooter - sellers raise prices of scooters by 50% then the number of scooters sold will decrease by $(0.25 \times 50\%) = 12.5\%$; but total revenue will rise by $(1 - 0.25) \times 50\% = 37.5\%$.

This problem can also be solved by using arc-price elasticity and the results will be about the same.

<2.10>

- A. Complements
- B. Complements
- C. Substitutes
- D. Substitutes
- E. Complements
- F. Complements (if sold together)

<2.11>

$$Q = \alpha P^\beta$$

$$12500 = \alpha(50)^{-0.75}$$

$$\alpha = 235,038$$

$$\begin{aligned} \text{An increase in fare from } 50\text{¢ to } 70\text{¢ will attract } Q &= 235,038(70)^{-0.75} \\ &= 9712 \text{ passengers} \end{aligned}$$

Revenue-wise we have

$$\text{At } 50\text{¢/ride} \times 12500 = \$6250$$

$$\text{At } 70\text{¢/ride} \times 9712 = \$6798$$

$$\text{Gain (total)} = \$548$$

Advice to management would be to raise fare to 70¢/ride

[Since $\epsilon = -0.75$, elastic, it is obvious what the conclusion would be]

<2.12>

(a)

$$\frac{\Delta Q/Q}{\Delta A/A} = -2.2 \Rightarrow 1\% \text{ reduction in travel time by auto}$$

will result in a 2.2% increase in automobile trips

$$\frac{\Delta Q/Q}{\Delta B/B} = 0.13 \Rightarrow 1\% \text{ reduction of travel time by bus will result in a } 0.13\% \text{ reduction in auto trips}$$

$$\frac{\Delta Q/Q}{\Delta C/C} = -0.4 \Rightarrow 1\% \text{ reduction in the avg cost of travel by auto will result in a } 0.4\% \text{ increase in auto trips}$$

$$\frac{\Delta Q/Q}{\Delta D/D} = 0.75 \Rightarrow 1\% \text{ reduction in the avg cost of travel by bus will result in a } 0.75\% \text{ reduction in auto trips}$$

The signs are justified.

- b) A = 20% ↗
 B = 10% ↗
 C = 5% ↗
 D = 15% ↘

$$Q_0 = aA^{-2.2}B^{-0.4}C^{-0.4}D^{0.75}$$

$$Q_1 = a(1.20A)^{-2.2}(1.10B)^{0.13}(1.05C)^{-0.4}(0.85D)^{0.75}$$

$$Q_1 = 0.589aA^{-2.2}B^{0.13}C^{-0.4}D^{0.75}$$

$$\frac{Q_1 - Q_0}{Q_0} = (0.589 - 1) = -0.411 \Rightarrow 41.1\% \text{ decrease in automobile trip}$$

- c) B = 10% ↘
 D = 10% ↗

$$Q_1 = aA^{-2.2}(0.9B)^{0.13}C^{-0.4}(1.10D)^{0.75} = 1.06$$

$$\frac{Q_1 - Q_0}{Q_0} = (1.06 - 1) = 0.06 \Rightarrow 6\% \text{ increase in automobile trip}$$

<2.13>

The cross elasticity coefficient indicates that the % change of express-bus riders (Q) will equal 2 times the % change in the price (P) of the ordinary bus

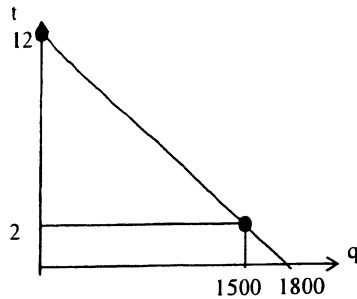
$$\epsilon = \frac{\% \Delta Q \text{ of express bus riders}}{\% \Delta P \text{ of ordinary bus riders}} = 2$$

$$\therefore \% \Delta Q \text{ of express bus riders} = 2 \times \% \Delta P \text{ of ordinary bus riders} = 2 \left[\frac{(P_2 - P_1)100}{(P_2 + P_1) / 2} \right]$$

$$= 2 \left[\frac{(0.5 - 0.75)100}{(0.5 + 0.75) / 2} \right] = -80\%$$

which means that express bus riders and revenue will decline by 80%, if the price of express bus service remains the same.

<2.14>



$$q = 1800 - 150t$$

$$\epsilon_t = \left(\frac{dq}{dt} \right) \left(\frac{t}{q} \right)$$

$$= (-150) \left(\frac{2}{1500} \right)$$

$$= -0.2 \text{ (inelastic)}$$

<2.15>

$$\frac{\partial Q}{Q} / \frac{\partial P}{P} = -0.75 \text{ (inelastic) or } \frac{20\%}{X\%} = -0.75, X\% = -26.7\%$$

a 26.7% decrease in fare = 73.3 cents and a new seating arrangement of 2400. Change in

$$\text{Consumer's surplus} = \frac{2000 + 2400}{2} \times 0.267 = \$587.4 \text{ per hour}$$

$$\text{Existing Revenue} = (2000)(1) = 2000$$

$$\text{Revised revenue} = (2400)(0.73) = 1760$$

$$\text{Loss} = \$240$$

Obviously, if the demand is sufficient, there is no need to decrease the fare.

<2.16>

Assume that the given price elasticity is valid in the range of the 10% change in the number of buses:

A 10% increase in the number of buses implies a 10% increase in the number of seats.

Price elasticity of - 0.3 means

$$\frac{\Delta Q}{Q} / \frac{\Delta P}{P} = -0.3$$

or a 3% increase in Q results from a 10% decrease in P. \therefore a 10% increase in Q would result in a 33.33% decrease in fare (P).

$$\text{Original capacity: } 50 \times 55 = 2750 \quad \text{New capacity: } 2750 + 275 = 3025$$