CHAPTER 1

Problem 1.1

Dollars deposited	= In
Checks written	= Out
Interest − Service charge	= Generation
Change in amount of dollars	= Rate of accumulation

Problem 1.2

(a) and (b) are unsteady, (c) is steady.

Problem 1.3

$$\frac{\partial}{\partial x}\left(\frac{\partial \varphi}{\partial y}\right) = 0$$

Integration with respect to x gives

$$\frac{\partial \varphi}{\partial y} = g(y)$$

Now, integration with respect to y gives

$$\varphi = \int g(y)\,dy + f(x) = h(y) + f(x) + C$$

where C is an integration constant.

Problem 1.4

From the conservation statement for energy

$$\begin{pmatrix}\text{Rate of}\\ \text{energy in}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \text{energy out}\end{pmatrix} = 2 \times 30 = 60\,\text{W}$$

$$(\text{Heat flux})_{\text{inner surface}} = \frac{60}{\pi(0.05)(30)} = 12.7\,\text{W/m}^2$$

$$(\text{Heat flux})_{\text{outer surface}} = \frac{60}{\pi(0.06)(30)} = 10.6\,\text{W/m}^2$$

We have obtained heat flux values differing by approximately 20% for the same heat flow rate.

Problem 1.5

Assumptions

1. The dust concentration is uniformly distributed throughout the foundry so that the air leaving the foundry has the same concentration as that of the air within the foundry.

2. Inlet air is dust-free.

The inventory rate equation for dust reduces to

$$\text{Rate of dust out} = \text{Rate of dust generation}$$

or,

$$\mathcal{Q}\,(20 \times 10^{-6}) = 0.3 \qquad \Rightarrow \qquad \mathcal{Q} = 15,000\,\mathrm{m}^3/\,\mathrm{h}$$

Problem 1.6

a) Let $\overline{x}$ and x be the coordinate systems in Figure (a) and (b), respectively. Note that

$$\overline{x} = x - B$$

Therefore, the velocity distribution for Figure (b) becomes

$$\begin{aligned} v_z &= \frac{|\Delta P|\,B^2}{2\,\mu L}\left[1 - \left(\frac{\overline{x}}{B}\right)^2\right] = \frac{|\Delta P|\,B^2}{2\,\mu L}\left[1 - \left(\frac{x-B}{B}\right)^2\right] \\ &= \frac{|\Delta P|\,B^2}{2\,\mu L}\left[2\left(\frac{x}{B}\right) - \left(\frac{x}{B}\right)^2\right] \end{aligned}$$

b) For Figure (a)

$$\mathcal{Q} = \int_0^W \int_{-B}^{B} v_z\,dx\,dy = \frac{2\,|\Delta P|\,B^3 W}{3\,\mu L}$$

For Figure (b)

$$\mathcal{Q} = \int_0^W \int_0^{2B} v_z\,dx\,dy = \frac{2\,|\Delta P|\,B^3 W}{3\,\mu L}$$

The volumetric flow rate is independent of the coordinate system.

Problem 1.7

The volumetric flow rate is given by

$$\mathcal{Q} = 2\int_0^H \int_0^{y/\sqrt{3}} v_z\,dx\,dy = 2\int_0^{H/\sqrt{3}} \int_{\sqrt{3}x}^{H} v_z\,dy\,dx = \frac{\sqrt{3}\,H^4\,|\Delta P|}{180\,\mu L}$$

Problem 1.8

The volumetric flow rate is given by

$$\mathcal{Q} = \int_0^{2\pi} \int_{-b}^{b} v_r\,r\,dz d\theta = 2\int_0^{2\pi} \int_0^{b} v_r\,r\,dz d\theta = 4\,\pi \int_0^b v_r\,r\,dz = \frac{4}{3}\,\frac{\pi b^3\,|\Delta P|}{\ln(R_2/R_1)}$$

CHAPTER 2

Problem 2.1

According to Newton's second law of motion

$$\text{Force} = \text{Time rate of change of momentum} \tag{1}$$

or,

$$\text{Force} = \frac{\text{Momentum}}{\text{Time}} \tag{2}$$

Therefore,

$$\frac{\text{Force}}{\text{Area}} = \frac{\text{Momentum}}{(\text{Area})\,(\text{Time})} = \text{Momentum Flux} \tag{3}$$

Problem 2.2

a) Equation (2.1-2) takes the form

$$\tau_{yx} = \frac{F}{A} = \mu\,\frac{dv_x}{dy} \tag{1}$$

or

$$\frac{F}{2}\int_0^{8\times10^{-3}} dy = 50\times10^{-3}\int_0^{0.4} dv_x \qquad \Rightarrow \qquad F = 5\,\text{N}$$

b) The shear stress is

$$\tau_{yx} = \frac{F}{A} = \frac{5}{2} = 2.5\,\text{Pa}$$

The use of Eq. (1) gives

$$2.5\int_0^{8\times10^{-3}} dy = 5\times10^{-3}\int_0^{V} dv_x \qquad \Rightarrow \qquad V = 4\,\text{m/s}$$

Problem 2.3

The shear stress τ_{yx} is constant everywhere in the fluid. At $y = Y_1$ the shear stress is continuous, i.e.,

$$(\tau_{yx})_A = (\tau_{yx})_B \qquad \text{at} \quad y = Y_1$$

or,

$$-\,\mu_A\left(\frac{0-2}{Y_1}\right) = -\,\mu_B\left(\frac{-1-0}{Y_2}\right)$$

Substitution of the numerical values gives

$$Y_2 = \frac{(0.8)(5)}{2} = 2\,\text{cm}$$

Problem 2.4

The rate of heat loss is expressed in the form

$$\dot{Q} = A \left(k \left. \frac{dT}{dx} \right|_{x=0} \right) = A \left(k \left. \frac{dT}{dx} \right|_{x=L} \right) \tag{1}$$

Note that the minus sign is omitted in Eq. (1) since temperature increases in the direction of x.

$$\frac{dT}{dx} = \text{constant} = 5\,^\circ\text{C/cm} = 500\,^\circ\text{C/m} \tag{2}$$

Therefore, the thermal conductivity is

$$k = \frac{72}{(3)(500)} = 0.048\,\text{W/m.K}$$

Problem 2.5

Since heat flux is a constant, integration of the Fourier's law of heat conduction gives

$$q_y \int_0^L dy = -\,k \int_{T_o}^{T_L} dT \qquad \Rightarrow \qquad q_y = \frac{k\,(T_o - T_L)}{L}$$

Substitution of the numerical values gives the heat flux as

$$q_y = \frac{(0.72)\,[30 - (-5)]}{0.2} = 126\,\text{W/m}^2$$

The rate of heat loss is

$$\dot{Q} = A\,q_y = (25)(126) = 3150\,\text{W}$$

Problem 2.6

The given temperature distribution indicates that temperature increases in the direction of z. The inventory rate equation for thermal energy gives

$$\text{Rate of energy generation} = \text{Rate of energy out} - \text{Rate of energy in}$$

or,

$$\Re\,AL = A \left(k \left. \frac{dT}{dz} \right|_{z=0} \right) - A \left(k \left. \frac{dT}{dz} \right|_{z=L} \right) \tag{1}$$

The temperature gradient is

$$\frac{dT}{dz} = 3000\,(1 - z) \tag{2}$$

Hence,

$$\left. \frac{dT}{dz} \right|_{z=0} = 3000\,^\circ\text{C/m} \qquad\qquad \left. \frac{dT}{dz} \right|_{z=L} = 3000\,(1 - 0.06) = 2820\,^\circ\text{C/m}$$

Therefore, the rate of generation per unit volume can be determined from Eq. (1) as

$$\Re = \frac{k}{A} \left(\left. \frac{dT}{dz} \right|_{z=0} - \left. \frac{dT}{dz} \right|_{z=L} \right) = \frac{15}{0.06}(3000 - 2820) = 45,000\,\text{W/m}^3 = 45\,\text{kW/m}^3$$

Problem 2.7

While both surfaces are at a temperature of 80 °C, the temperature at the center is greater than 80 °C. Therefore, heat is transferred from the wall to the surroundings through both surfaces, and it is expressed in the form

$$\dot{Q} = A\left(q_z|_{z=0} + q_z|_{z=L}\right) = \frac{Ak}{L}\left(\left.\frac{\partial T}{\partial \xi}\right|_{\xi=0} - \left.\frac{\partial T}{\partial \xi}\right|_{\xi=1}\right) \tag{1}$$

The temperature gradients are calculated as

$$\left.\frac{\partial T}{\partial \xi}\right|_{\xi=0} = \frac{10\,\pi}{L}\,e^{-0.09t} \qquad \left.\frac{\partial T}{\partial \xi}\right|_{\xi=1} = -\,\frac{10\,\pi}{L}\,e^{-0.09t} \tag{2}$$

The use of Eq. (2) in Eq. (1) leads to

$$\dot{Q} = \frac{20\,\pi A\,k}{L}\,e^{-0.09t} \tag{3}$$

The amount of heat transferred is

$$Q = \int_0^t \dot{Q}\,dt = \frac{20\,\pi Ak}{0.09\,L}\left(1 - e^{-0.09t}\right) \tag{4}$$

Substitution of the numerical values into Eq. (4) gives

$$Q = \frac{(20)\pi(15)(20)}{(0.09)(0.6)}\left[1 - e^{-(0.09)(0.5)}\right] = 15,360\,\mathrm{J}$$

Problem 2.8

Since temperature reaches its maximum value within the wall, energy is lost from both surfaces. Considering wall as the system, the inventory rate equation for thermal energy gives

$$\text{Rate of energy generation} = \text{Rate of energy out} \tag{1}$$

Equation (1) can be expressed in symbolic form as

$$\Re\,AL = A\left(k\left.\frac{dT}{dz}\right|_{z=0} - k\left.\frac{dT}{dz}\right|_{z=1}\right) \tag{2}$$

or,

$$\Re = \frac{k}{L}\left(\left.\frac{dT}{dz}\right|_{z=0} - \left.\frac{dT}{dz}\right|_{z=1}\right) \tag{3}$$

Assume that the variation of temperature as a function of position is given by

$$T = a\,z^2 + b\,z + c$$

The use of Eqs. (A.6-15)-(A.6-17) in Appendix A gives

$$T = -\,120\,z^2 + 170\,z + 30 \tag{4}$$

Therefore,

$$\left.\frac{dT}{dz}\right|_{z=0} = 170\,{}^\circ\mathrm{C/\,m} \qquad \left.\frac{dT}{dz}\right|_{z=1} = -\,70\,{}^\circ\mathrm{C/\,m} \tag{5}$$

The use of Eq. (5) in Eq. (3) leads to

$$\Re = \frac{8}{1}(170 + 70) = 1920\,\mathrm{W/\,m^3}$$

Problem 2.9

a) The rate of heat loss is given by

$$\begin{aligned}\dot{Q} &= \text{(Thermal conductivity)(Geothermal gradient)(Area)} \\ &= (3 \times 1000)(25)\left[\pi(1.27 \times 10^4)^2\right] = 38 \times 10^{12}\,\text{W} = 38 \times 10^9\,\text{kW}\end{aligned}$$

b) Rate of heat loss through $1\,\text{m}^2$ is

$$\dot{Q} = (3)(0.025)(1) = 0.075\,\text{W}$$

Amount of heat loss in 4 days is

$$Q = (0.075)(3600)(24)(4) = 25,920\,\text{J}$$

Let us assume that the mass of coffee in a cup is 200 g. Taking the heat capacity as 4.2 J/ g. K

$$25,920 = (200)(4.2)(T - 20) \qquad \Rightarrow \qquad T \simeq 51\,^\circ\text{C}$$

Problem 2.10

The temperature gradient at $z = 0$ is

$$\left.\frac{\partial T}{\partial z}\right|_{z=0} = \frac{T_o}{\sqrt{\pi\,\alpha\,t}} = 0.025\,^\circ\text{C/ m} \tag{1}$$

Solving for time yields

$$t = \frac{1}{\pi\,\alpha}\left(\frac{T_o}{0.025}\right)^2 \tag{2}$$

The thermal diffusivity is

$$\alpha = \frac{k}{\rho\,\widehat{C}_P} = \frac{3}{(5500)(2000)} = 272.7 \times 10^{-9}\,\text{m}^2/\,\text{s} \tag{3}$$

Therefore, the time needed for the geothermal gradient to fall its present value, i.e., the age of the Earth, is

$$t = \frac{1}{\pi(272.7 \times 10^{-9})}\left(\frac{1200}{0.025}\right)^2 = 2.7 \times 10^{15}\,\text{s} = 85.3 \times 10^6\ \text{year}$$

Problem 2.11

$$\dot{Q} = -\,Ak\left.\frac{\partial T}{\partial z}\right|_{z=0} \tag{1}$$

Application of the Leibnitz's rule gives

$$-\frac{\partial T}{\partial z} = \frac{2(T_1 - T_o)}{\sqrt{\pi}}\,\frac{1}{2\sqrt{\alpha\,t}}\,\exp\left(-\frac{z^2}{4\,\alpha\,t}\right)$$

or,

$$-\left.\frac{\partial T}{\partial z}\right|_{z=0} = \frac{(T_1 - T_o)}{\sqrt{\pi\,\alpha\,t}} \tag{2}$$

Substitution of Eq. (2) into Eq. (1) gives the rate of heat transferred into the slab as

$$\dot{Q} = \frac{Ak\,(T_1 - T_o)}{\sqrt{\pi\,\alpha\,t}} \tag{3}$$

Amount of heat transferred is

$$Q = \int_0^t \dot{Q}\,dt = \frac{2\,Ak\,(T_1 - T_o)}{\sqrt{\pi\,\alpha}}\,\sqrt{t} \tag{4}$$

Problem 2.12

Mass diffusivity of ethanol in air at 20 °C (293 K) is

$$(\mathcal{D}_{AB})_{293} = (\mathcal{D}_{AB})_{313}\left(\frac{293}{313}\right)^{3/2} = (1.45 \times 10^{-5})\left(\frac{293}{313}\right)^{3/2} = 1.31 \times 10^{-5}\,\mathrm{m^2/\,s}$$

The molar flux is

$$N_{A_z}\big|_{z=0} = -\,\mathcal{D}_{AB}\,\frac{dc_A}{dz}\bigg|_{z=0} = (1.31 \times 10^{-5})(6)(3600) = 0.283\,\mathrm{kmol/\,m^2.\,h}$$

Problem 2.13

Since $V = n/c$, the definition of the partial molar volume becomes (note that $c =$ constant)

$$\overline{V}_i = \frac{1}{c}\left[\frac{\partial}{\partial n_i}(n_1 + n_2 + ... + n_i + ...)\right]_{T,P,n_{j\neq i}} = \frac{1}{c} \tag{1}$$

Therefore,

$$c_i\,\overline{V}_i = \frac{c_i}{c} = x_i \tag{2}$$

indicating that the volume fraction is equal to the mole fraction for constant total molar concentration.

Problem 2.14

The Schmidt number is

$$\mathrm{Sc} = \frac{\mu}{\rho\,\mathcal{D}_{AB}} \tag{1}$$

Since

$$\mathcal{D}_{AB} \propto \frac{T^{3/2}}{P} \qquad \text{and} \qquad \rho = \frac{P\mathcal{M}}{\mathcal{R}T} \tag{2}$$

Eq. (1) implies that

$$\mathrm{Sc} \propto \frac{\mu\,\mathcal{R}}{\mathcal{M}\sqrt{T}}$$

Problem 2.15

Assumptions

1. Molecular flux $\gg$ Convective flux (Reasonable assumption within the liquid phase)
2. $c =$ constant

$$N_{A_z}\big|_{z=0} = -\mathcal{D}_{AB}\,\frac{dc_A}{dz}\bigg|_{z=0} = \frac{\mathcal{D}_{AB}\,c_{A_o}\Lambda\tanh\Lambda}{L}$$

Note that the molar flux is independent of position. Therefore, the molar rate of species $\mathcal{A}$ entering into the liquid is

$$\dot{n}_A = A\;N_{A_z}\big|_{z=0} = \frac{A\mathcal{D}_{AB}\,c_{A_o}\Lambda\tanh\Lambda}{L}$$

The molar flux at the bottom of the container is

$$N_{A_z}\big|_{z=L} = -\mathcal{D}_{AB}\,\frac{dc_A}{dz}\bigg|_{z=L} = 0$$

indicating that the species $\mathcal{A}$ cannot diffuse through the bottom of the container (impermeable surface).