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PROBLEM 1.1

Using the continuity equation, let's first verify that the given velocities satisfy local mass conservation in the given incompressible flow field.

Continuity equation: $$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}+\frac{\partial V_z}{\partial z}=0$$

$$\frac{\partial(5xe^{-2z})}{\partial x}+\frac{\partial(-3ye^{-2z})}{\partial y}+\frac{\partial(e^{-2z})}{\partial z}$$

$$5e^{-2z}-3e^{-2z}-2e^{-2z}=0$$

Streamline equation: $$\frac{dx}{V_x}=\frac{dy}{V_y}=\frac{dz}{V_z}$$

$$\frac{dx}{5xe^{-2z}}=\frac{dy}{-3ye^{-2z}}=\frac{dz}{e^{-2z}}$$

$$\frac{dx}{5x}=-\frac{1}{3}\frac{dy}{y}=dz$$

Integration of the first two terms yields

$$\frac{1}{5}\ln x=-\frac{1}{3}\ln y+\ln C_1'$$

$$y=C_1x^{-3/5}$$

Since the streamline passes through the point $A(1,1,1)$, we obtain $C_1=1$, giving $y=x^{-3/5}$. Similarly, the integration of $dx/5x=dz$ yields

$$\frac{1}{5}\ln x=z+C_2$$

$$z=\frac{1}{5}\ln x-C_2$$

For the streamline to pass through the point $A(1,1,1)$, we must have $C_2=-1$, giving

$$z=1+\frac{1}{5}\ln x$$

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Thus, in the given flow field, the streamline passing through the point $A(1,1,1)$ is given by the equations:

$$\boxed{y = x^{-3/5}} \quad \text{and} \quad \boxed{z = 1 + \frac{1}{5}\ln x}$$

PROBLEM 1.2

The given flow field must satisfy the continuity equation with variable density. Let's assume that the new velocity in the z-direction has the form $V_z = ae^{-bz}$ where a and b are constants.

Continuity equation:

$$\frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} + \frac{\partial(\rho V_z)}{\partial z} = 0$$

$$\frac{\partial(5xe^{-2z}e^{-z})}{\partial x} + \frac{\partial(-3ye^{-2z}e^{-z})}{\partial y} + \frac{\partial(ae^{-bz}e^{-z})}{\partial z} = 0$$

$$5e^{-3z} - 3e^{-3z} - a(1+b)e^{-(1+b)z} = 0$$

From the above equation we obtain $b = 2$ and $a = 2/3$, giving $V_z = 2/3e^{-2z}$.

Streamline equation:

$$\frac{dx}{V_x} = \frac{dy}{V_y} = \frac{dz}{V_z}$$

The above equation for the streamline depends only upon the velocity field. The influence of density variation on streamlines occurs through the altered velocity field, which must satisfy the local mass conservation.

$$\frac{dx}{5xe^{-2z}} = \frac{dy}{-3ye^{-2z}} = \frac{3}{2}\frac{dz}{e^{-2z}}$$

$$\frac{dx}{5x} = \frac{1}{3}\frac{dy}{y} = \frac{3}{2}dz$$

Integration of the first two terms yields the same solution as obtained for Problem 1.1

$$y = x^{-3/5}$$

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Integration of $\frac{dx}{5x} = \frac{3}{2}dz$ yields

$$\frac{1}{5}\ln x = \frac{3}{2}z + C_2'$$

$$z = \frac{2}{15}\ln x - C_2$$

For the streamline to pass through the point $A(1,1,1)$, we must have $C_2 = -1$, giving

$$z = 1 + \frac{2}{15}\ln x$$

Thus, the streamline passing through the point $A(1,1,1)$ in the flow field with the given density variation is represented by the equations:

$$\boxed{y = x^{-3/5}} \qquad \text{and} \qquad \boxed{z = 1 + \frac{2}{15}\ln x}$$

PROBLEM 1.3

The two-dimensional continuity equation in the boundary layer is expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Substituting for $u = U\left[2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2\right]$ in the first term of the above equation yields

$$\frac{\partial u}{\partial x} = U\left[-2\frac{y}{\delta^2} + 2\frac{y^2}{\delta^3}\right]\left(\frac{K}{2\sqrt{x}}\right) = \frac{KU}{\sqrt{x}}\left[-\frac{y}{\delta^2} + \frac{y^2}{\delta^3}\right]$$

Thus, the continuity equation yields

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = \frac{KU}{\sqrt{x}}\left[\frac{y}{\delta^2} - \frac{y^2}{\delta^3}\right]$$

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$$v = \frac{KU}{\sqrt{x}}\left[\frac{y^2}{2\delta^2} - \frac{y^3}{3\delta^3}\right] + C$$

where C is the integration constant. Since $v = 0$ at $y = 0$ (impermeable flat plate), we obtain $C = 0$, giving

$$\boxed{v = \frac{KU}{\sqrt{x}}\left[\frac{y^2}{2\delta^2} - \frac{y^3}{3\delta^3}\right]}$$

PROBLEM 1.4

Diffuser diameter: $$D_x = D_1 + x\left(\frac{D_2 - D_1}{L}\right) = D_1 + \alpha x$$

where $\alpha = \left(\frac{D_2 - D_1}{L}\right)$

Diffuser flow area: $$A_x = \frac{\pi D_x^2}{4} = \frac{\pi}{4}(D_1 + \alpha x)^2$$

Velocity at x: $$V_x = \frac{Q}{A_x} = \frac{4Q}{\pi}\frac{1}{(D_1 + \alpha x)^2}$$

Convective acceleration: $$a_x = V_x \frac{dV_x}{dx}$$

$$\frac{dV_x}{dx} = -\frac{8\alpha Q}{\pi}\frac{1}{(D_1 + \alpha x)^3}$$

Thus, we obtain

$$a_x = -\left[\frac{4Q}{\pi}\frac{1}{(D_1 + \alpha x)^2}\right]\left[\frac{8\alpha Q}{\pi}\frac{1}{(D_1 + \alpha x)^3}\right]$$

$$\boxed{a_x = -\left(\frac{32\alpha Q^2}{\pi^2}\right)\frac{1}{(D_1 + \alpha x)^5}}$$

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PROBLEM 1.5

Velocity components and their partial derivatives:

$$V_x = x^2y + 5xt + yz^2\,;\ \frac{\partial V_x}{\partial t} = 5x\,;\ \frac{\partial V_x}{\partial x} = 2xy + 5t\,;\ \frac{\partial V_x}{\partial y} = x^2 + z^2\,;\ \frac{\partial V_x}{\partial z} = 2yz$$

$$V_y = xy^2 + 4t\,;\ \frac{\partial V_y}{\partial t} = 4\,;\ \frac{\partial V_y}{\partial x} = y^2\,;\ \frac{\partial V_y}{\partial y} = 2xy\,;\ \frac{\partial V_y}{\partial z} = 0$$

$$V_z = -(4xyz - xy + 5zt)\,;\ \frac{\partial V_z}{\partial t} = -5z\,;\ \frac{\partial V_z}{\partial x} = -4yz + y\,;\ \frac{\partial V_z}{\partial y} = -4xz + x\,;\ \frac{\partial V_z}{\partial z} = -4xy - 5t$$

(a) Continuity equation: $\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0$

$$2xy + 5t + 2xy - 4xy - 5t = 4xy - 4xy + 5t - 5t = 0$$

(b) Acceleration vector: $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$$a_x = \frac{\partial V_x}{\partial t} + V_x\frac{\partial V_x}{\partial x} + V_y\frac{\partial V_x}{\partial y} + V_z\frac{\partial V_x}{\partial z}$$

At $A(1,1,1)$: $\frac{\partial V_x}{\partial t} = 5x = 5$

$$V_x\frac{\partial V_x}{\partial x} = (x^2y + 5xt + yz^2)(2xy + 5t) = 49$$

$$V_y\frac{\partial V_x}{\partial y} = (xy^2 + 4t)(x^2 + z^2) = 10$$

$$V_z\frac{\partial V_x}{\partial z} = (4xyz - xy + 5zt)(4xy + 5t) = 72$$

$$a_x = 5 + 49 + 10 + 72 = 136$$

$$\frac{\partial V_y}{\partial t} = 4$$

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$$V_x \frac{\partial V_y}{\partial x} = (x^2 y + 5xt + yz^2)(y^2) = 7$$

$$V_y \frac{\partial V_y}{\partial y} = (xy^2 + 4t)(2xy) = 10$$

$$V_z \frac{\partial V_y}{\partial z} = (4xyz - xy + 5zt)(0) = 0$$

$$a_y = 4 + 7 + 10 + 0 = 21$$

$$\frac{\partial V_z}{\partial t} = -5z = -5$$

$$V_x \frac{\partial V_z}{\partial x} = (x^2 y + 5xt + yz^2)(-4yz + y) = -21$$

$$V_y \frac{\partial V_z}{\partial y} = (xy^2 + 4t)(-4xz + x) = -15$$

$$V_z \frac{\partial V_z}{\partial z} = (4xyz - xy + 5zt)(-4xy - 5t) = -72$$

$$a_z = -5 - 21 - 15 - 72 = -113$$

$$\boxed{\vec{a} = 136\hat{i} + 21\hat{j} - 113\hat{k}}$$

(c) Rotation vector: $\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$

At $A(1,1,1)$: $\omega_x = \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z} = -4xz + x - 0 = x(1 - 4z) = -3$

$$\omega_y = \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} = 2yz + 4yz = 6$$

$$\omega_z = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} = y^2 - x^2 - z^2 = -1$$

$$\boxed{\vec{\omega} = -3\hat{i} + 6\hat{j} - \hat{k}}$$

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The results show that the rotation vector or the vorticity vector, which is twice the rotation vector, is steady everywhere in the given unsteady three-dimensional velocity field.

PROBLEM 1.6

As shown in the worked-out Example 1.5, the vorticity in a forced vortex has a constant value of 2Ω everywhere. The circulation for any closed curve in a forced vortex equals the product of its enclosed area and the uniform value of vorticity (2Ω). Accordingly, the circulation around both squares A and B will be $2\Omega a^2$.

PROBLEM 1.7

As shown in the worked-out Example 1.5, the circulation for any closed curve that includes the origin has a constant value of $2\pi C$ and it is zero outside the curve. Accordingly, the circulation around the square A is $2\pi C$ and that for square B is zero.