Conditions at 1
\( p_1 = 0.5 \text{bar}; \ t_1 = 81.3^\circ \text{C}; \ \rho_{g1} = 3.239 \text{m}^3/\text{kg} \)
\( h_{f1} = 340 \text{kJ/kg}; \ h_{g1} = 2645 \text{kJ/kg}; \ s_{f1} = 1.091 \text{kJ/kgK}; \ s_{g1} = 7.593 \text{kJ/kgK} \)

Conditions at 2
\( p_2 = 20 \text{bar}; \ t_2 = 212.4^\circ \text{C}; \ \rho_{g2} = 0.09957 \text{m}^3/\text{kg} \)
\( h_{f2} = 909 \text{kJ/kg}; \ s_{f2} = 2.447 \text{kJ/kgK}; \ s_{g2} = 6.340 \text{kJ/kgK} \)

Conditions at 3
\( p_3 = 20 \text{bar}; \ \rho_3 = 6.340 \text{kJ/kgK} \)
\( h_3 = 2799 \text{kJ/kg} \)

Energy added: \( q_{23} = h_3 - h_2 = 2799 - 909 = 1890 \text{kJ/kg} \)

Conditions at 4
\( p_4 = 0.5 \text{bar}; \ s_4 = s_3 = 6.340 \text{kJ/kgK}, \ \text{thus} \ x_4 = \frac{6.340 - 1.091}{7.593 - 1.091} = 0.8073 \)
\( h_4 = x_4 h_f + (1-x) h_g = 0.8073 \times 2645 + (1 - 0.8073) \times 340 = 2200.8 \text{kJ/kg} \)

Conditions at 1
\( p_1 = 0.5 \text{bar}; \ s_1 = s_2 = 2.447 \text{kJ/kgK}, \ \text{thus} \ x_1 = \frac{2.447 - 1.091}{7.593 - 1.091} = 0.2086 \)
\( h_1 = x_1 h_f + (1-x) h_g = 0.2086 \times 2645 + (1 - 0.2086) \times 340 = 820.8 \text{kJ/kg} \)
\( q_{41} = h_4 - h_1 = -1380 \text{kJ/kg} \)
\( w_{net} = 510 \text{kJ/kg} \)
\( \eta_m = 510/1890 = 0.2698 = 26.98\% \)

Carnot efficiency, \( \eta_m = 1 - \frac{T_c}{T_1} = 1 - \frac{354.3}{485.4} = 0.2700 = 27.0\% \)
Chapter 3 Solutions

P3.2

Conditions at 1

\( p_1 = 0.5 \text{bar}; \; t_1 = 81.3^\circ \text{C}; \; v_{g1} = 3.239 \text{m}^3/\text{kg} \)

\( h_{f1} = 340 \text{kJ/kg}; \; h_{g1} = 2645 \text{kJ/kg}; \; s_{f1} = 1.091 \text{kJ/kgK}; \; s_{g1} = 7.593 \text{kJ/kgK} \)

Conditions at 3

\( p_3 = 20 \text{bar}; \; t_3 = 212.4^\circ \text{C}; \; v_{g3} = 0.09957 \text{m}^3/\text{kg} \)

\( h_{f3} = 909 \text{kJ/kg}; \; h_{g3} = 2.447 \text{kJ/kgK}; \; s_{g3} = 6.340 \text{kJ/kgK} \)

Many conditions remain the same as in P3.1: note change in state point numbering.

Feed pump work, \( w_p = w_{f2} = -\bigtriangleup p \cdot v = \left\{ 0.1029 + \frac{0.5 - 0.4736}{0.578 - 0.4736} \times 0.0003 \right\} \times 10^{-2} = 0.1030 \times 10^{-2} \text{ m}^3 / \text{kg} \).

Hence, \( w_p = -0.1030 \times 10^{-2} \times (20.0 - 0.5) = -2 \text{kJ/kg} \).

Energy added, \( q_{in} = h_4 - h_2 = 2799 - 342 = 2457 \text{kJ/kg} \).

Energy rejected, \( q_{out} = h_1 - h_3 = 340 - 2200.8 = -1861 \text{kJ/kg} \).

Thus efficiency, \( \eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{1861}{2457} = 24.25\% \).

Alternative calculation, \( \eta_{th} = \frac{w_{net}}{q_{in}} = \frac{596}{2457} = 0.2425 = 24.25\% \).

Back work ratio

\[
\text{Rankine } r_{wb} = \frac{w_{p}}{w_{r}} = \frac{2}{598} = 0.0034 = 0.34\%
\]

\[
\text{Carnot } r_{wb} = \frac{w_{p}}{w_{r}} = \frac{88}{598} = 0.147 = 14.7\%
\]

The high value of \( r_{wb} \) for the Carnot cycle shows that it is very susceptible to inefficiencies in the pump, and turbine.
P3.3
Recalculating P3.1 with $\eta_p = 0.8$ and $\eta_T = 0.9$

\[ w_p = \frac{88.2}{0.8} = -110.25 \text{kJ/kg}; \quad w_T = 0.9 \times 598 = 538.2 \text{kJ/kg} \]

\[ T_1 = 10.25 \text{kJ/kg}, \quad T_2 = 0.9 \times 598 = 538.2 \text{kJ/kg} \]

\[ Q_{in} = 538.2 - 110.25 = 22.64\% \]

Thus

\[ r_{wb} = \frac{110.25}{538.2} = 20.48\% \]

P3.4
Recalculating P3.2 using $\eta_p = 0.8$ and $\eta_T = 0.9$

\[ w_p = \frac{2}{0.8} = -2.5 \text{kJ/kg}; \quad w_T = 0.9 \times 598 = 538.2 \text{kJ/kg} \]
Chapter 3 Solutions

\[ \eta_{th} = \frac{538.2 - 2.5}{2799 - 342.5} = 21.81\% \]

Thus \[ r_{wb} = \frac{2.5}{538.2} = 0.4645\% \]

P3.5 Lenoir engine cycle

Conditions at 1 \[ p_1 = 1\text{bar}; t_1 = 15\degree\text{C}; T_1 = 288\text{K} \]

Conditions at 2 \[ p_2 = 10\text{bar}; T_2 = 2880\text{K} \]

Volume at 1, \[ V_1 = \left(0.287 \times 10^3 \times 288\right) / \left(1 \times 10^3\right) = 0.82656\text{m}^3 / \text{kg} \]

Expansion ratio, \[ r_e = V_3 / V_1 = V_4 / V_1 = 5; p_3 = 1.0506\text{bar}. \]

Work done in expansion 2-3 \[ w_{23} = \frac{p_2V_2 - p_3V_3}{K - 1} = \frac{\left(10 \times 0.82656 - 1.0506 \times 4.1328\right)}{0.4} \times 10^3 \]

\[ = 980.92 \text{kJ/kg} \]

Conditions at 4 \[ p_4 = 1.0\text{bar}; V_4 = 5V_1 \]

Work done in compression back to 1, \[ w = 1.0 \times \left(0.82656 - 4.1328\right) \times 10^3 = -330.62\text{kJ/kg} \]

Total work, \[ w_{net} = 980.92 - 330.62 = 650.30 \text{kJ/kg} \]

Energy addition, \[ q_{in} = c_p(T_2 - T_1) = 0.7175 \times (2880 - 288) = 1859.76 \text{kJ/kg} \]

Thermal efficiency, \[ \eta_{th} = \frac{w_{net}}{Q_{in}} = \frac{650.30}{1859.76} = 0.3497 \text{ (34.97\%)} \]

P3.6
Chapter 3 Solutions

Conditions at 1 $p_1 = 1$ bar; $t_1 = 27^\circ$C; $T_1 = 300$ K

Energy added, $q_{in} = q_{12} = 1000$ kJ/kg

$T_2 = T_1 + \frac{q_{12}}{c_v} = 300 + \frac{1000}{0.717} = 1694.7$ K

Conditions at 2 $p_2 = 5.65$ bar

Expansion ratio, $\gamma_e = V_3/V_1 = V_4/V_1 = 3$; $p_3 = \frac{5.65}{3} = 1.21$ bar.

$$w_{23} = \frac{p_2 v_2 - p_3 v_3}{\kappa - 1} = \left(\frac{5.65 \times 1.0 - 1.21 \times 3.0}{0.4}\right) \times \frac{287 \times 300}{1.0 	imes 10^5} \times \frac{10^5}{10^3} p_1 = 343.22 \text{ kJ/kg}$$

Conditions at 4 $p_4 = 1.0$ bar; $V_4 = 3V_1$

Work done in compression back to 1, $w = 1.0 \times (1.0 - 3.0) \times \frac{RT_1}{p_1} \times 10^2 = -172.2$ kJ/kg

Total work, $w_{net} = 432.22 - 172.2 = 260.02$ kJ/kg

Thermal efficiency, $\eta_{th} = \frac{w_{net}}{Q_{in}} = \frac{260.02}{1000.00} = 0.26002$ (26.00%)

P3.7

$$w_{net} = w_{23} + w_{31}$$

$$w_{23} = \frac{p_2 v_2 - p_3 v_3}{\kappa - 1} = \frac{RT_2 - T_3}{\kappa - 1}$$

Net work done,

$$w_{31} = \frac{1}{3} p dV = p_3 V_1 \ln \left(\frac{V_1}{V_3}\right) = RT_1 \ln \left(\frac{V_2}{V_3}\right) = RT_1 \ln \left(\frac{T_2}{T_3}\right)^{\kappa - 1}$$

$$w_{net} = \frac{RT_2 - T_3}{\kappa - 1} + RT_1 \left(\frac{T_2}{T_3}\right)^{\kappa - 1}$$

Energy addition, $q_{in} = c_v (T_2 - T_1) = \frac{RT_2 - T_1}{\kappa - 1}$
Hence, \( \eta_{th} = 1 - \frac{T_1}{T_2 - T_1} \ln \frac{T_2}{T_1} \)

P3.8

Energy added: \( q_{12} = c_v(T_2 - T_1) \)

Energy rejected: \( q_{31} = c_p(T_1 - T_3) \)

Net work: \( w_{net} = c_v(T_2 - T_1) + c_p(T_1 - T_3) \)

Thermal efficiency: \( \eta_{th} = \frac{w_{net}}{q_{in}} = \frac{c_v(T_2 - T_1) + c_p(T_1 - T_3)}{c_v(T_2 - T_1)} = 1 - \frac{\kappa(T_3 - T_1)}{(T_2 - T_1)} \)

\[
\begin{align*}
  w_{23} &= \frac{p_2 v_2 - p_3 v_3}{\kappa - 1} = \frac{R(T_2 - T_3)}{\kappa - 1} \\
  w_{31} &= p_1(v_1 - v_3) = RT_1(1 - T_3/T_1)
\end{align*}
\]

Calculate around cycle:

\( p_1 = 1 \text{bar}; T_1 = 27{\degree}\text{C} = 300\text{K} \)

Energy added, \( q_{in} = 2000\text{kJ/kg} \)

\( T_2 = T_1 + \frac{q_{in}}{c_v} = 300 + \frac{2000}{0.718} = 3085\text{K} \)

\( p_2 = p_1 \left( \frac{T_2}{T_1} \right) = 1 \times \left( \frac{3085}{300} \right) = 10.28\text{bar} \)

\( T_3 = T_2 \left( \frac{p_3}{p_2} \right)^{(\kappa-1)/\kappa} = 3085 \times \left( \frac{10.28}{10.28} \right)^{0.286} = 1584\text{K} \)

Hence, \( \eta_{th} = 1 - \frac{1.4 \times (1584 - 300)}{(3085 - 300)} = 35.45\% \)

\( w_{net} = \eta_{th} q_{in} = 0.3545 \times 2000 = 709\text{kJ/kg} \)

The following solutions will just give the values at the salient points: no calculations will be shown. The Rankine cycles applicable to the early solutions are shown below.
Chapter 3 Solutions

Conditions at 1.
\[ p_1 = 0.15 \text{ bar}; \quad t_1 = 53.98^\circ C; \quad \rho_1 = 10.02 \text{ m}^3/\text{kg}; \quad \rho_{f1} = 0.1014 \times 10^2 \]
\[ h_f = 226 \text{ kJ/kg}; \quad h_{g1} = 2598 \text{ kJ/kg}; \quad \rho = 0.7551 \text{ kJ/kgK}; \quad \rho_1 = 8.01 \text{ kJ/kgK}. \]

Conditions at 3.
\[ p_3 = 0 \text{ bar}; \quad t_3 = 212.4^\circ C; \quad \rho_3 = 0.00857 \text{ m}^3/\text{kg}; \quad h_f = 903 \text{ kJ/kg}; \quad \rho_f = 2.447 \text{ kJ/kgK}; \quad \rho_3 = 6.340 \text{ kJ/kgK}. \]

Feed pump work \[ \rho_p = \rho_1 = -V dp \]
\[ V = 0.1014 + \frac{0.15 - 0.1233}{0.1574 - 0.1233} = 0.10143 \times 10^{-2} \text{ m}^3/\text{kg}. \]

Thus \[ \rho_p = \rho_1 = -V dp = -0.001013 \times 0.030 = 2.815 \text{ kJ/kg}. \]

Hence, \[ h_2 = h_f + 2 = h_f + 228 \text{ kJ/kg}. \]

Conditions at 4
\[ p_4 = 0 \text{ bar}; \quad t_4 = 212.4^\circ C; \quad \rho_4 = 0.340 \text{ kJ/kgK}; \quad h_f = 0.798 \text{ kJ/kg}. \]

Conditions at 6
\[ p_6 = 0.15 \text{ bar}; \quad \rho_6 = 8.01 \text{ kJ/kgK}; \quad \rho_6 = 0.7551 \text{ kJ/kgK}. \]
\[ t_6 = 53.98^\circ C; \quad h_f = 226 \text{ kJ/kg}; \quad h_{g6} = 2598 \text{ kJ/kg}. \]

Thus \[ x_6 = \frac{6.340 - 0.7551}{8.01 - 0.7551} = 0.7698 \]
giving \[ h_6 = h_f + x_6(h_{g6} - h_f) = h_f + 0.7698(2598 - 226) \]
\[ = 2552 \text{ kJ/kg}. \]

Net work done by turbine plant \[ \rho_{net} = (h_f - h_f) + (h_f - h_2) \]
\[ = (2552 - 2052) - 2 = 744 \text{ kJ/kg}. \]

Energy added \[ q_{in} = (h_f - h_2) = 2799 - 228 = 2571 \text{ kJ/kg}. \]

Hence \[ \rho_{th} = \frac{q_{net}}{q_{in}} = \frac{744}{2571} = 28.94\% \]

Could also be calculated as \[ \rho_{th} = 1 - \frac{q_{out}}{q_{in}} \]
\[ \text{then} \quad \rho_{th} = 1 - \frac{h_f - h_f}{q_{in}} = 1 - \frac{1886}{2571} = 28.97\% \]

The Carnot cycle efficiency between these two temperatures would be \[ \rho_{Carn} = \frac{212.4 - 53.9}{212.4 + 273} = 32.65\% \]
P3.10. In this question the steam is superheated to 400°C in an effort to increase the dryness at point 6.

Conditions at state points 1, 2, and 3 are same as in P3.9

Conditions at 5 are superheated

\( p_5 = 0.06 \text{bar}; \quad t_5 = 400\,^\circ\text{C}; \quad h_5 = 3248 \text{kJ/kg}; \quad s_5 = 7.126 \text{kJ/kg} \)

Conditions at 6 \( p_6 = 0.15 \text{bar}; \quad t_6 = 53.9\,^\circ\text{C} \)

\[ x_6 = \frac{7.126 - 0.755}{8.01 - 0.755} = 0.8782 \text{ (dryness has been improved)} \]

\[ h_6 = h_{ce} + x_6 (h_{g6} - h_{f6}) = 226 + 0.8782 \times (2538 - 226) = 2309 \text{ kJ/kg} \]

Hence net work output, \( W_{net} = (h_5 - h_6) + (h_1 - h_2) \)

\[ = (3248 - 2309) - 2 = 937 \text{ kJ/kg} \]

Energy added to working fluid

\[ q_{in} = h_5 - h_2 = 3248 - 226 = 3020 \text{ kJ/kg} \]

Hence \( \eta_{th} = \frac{937}{3020} = 0.3102 = 31.02\% \)

Carrot efficiency for these temperatures

\[ \eta_{Carr} = 1 - \frac{T_c}{T_h} = 1 - \frac{326.9}{673} = 0.5142 = 51.42\% \]

Mean temperature of heat (energy) addition

\[ T_h = \frac{h_5 - h_2}{s_5 - s_2} = \frac{3248 - 226}{7.126 - 0.755} = 474 \text{ K} \]

Mean temperature of heat (energy) rejection

\[ T_c = \frac{58.9 + 326.9}{2} = 192.9 \text{ K} \]

Thus Carrot efficiency based on \( T_h \) and \( T_c \) is

\[ \eta = \frac{1 - \frac{T_c}{T_h}}{1 - \frac{326.9}{474}} = 31.73\% \]

Hence, Carrot efficiency based on mean temperatures is same as actual cycle efficiency
The front end of the cycle to state pt 4 and 5 is same as in P3.10.

State points G to 8 are different.

Expansion 5 to 6 is from 2obar to 1obar.

Hence state point 6 is $P_6 = 1$obar; $s_6 = 7.126 \text{ kJ/kg K}$.

Thus state point 6 is still superheated and $t_6 = 300^\circ\text{C}$

Hence, $h_6 = 3052 \text{ kJ/kg}$

and $w_{5G} = 3248 - 3052 = 196 \text{ kJ/kg}$

Reheat along isobar from 6 to 7 at 1obar.

State point 7, $P_7 = 1$obar; $t_7 = 400^\circ\text{C}$

$h_7 = 3264 \text{ kJ/kg}$; $s_7 = 7.464 \text{ kJ/kg K}$

Hence, $x_8 = \frac{7.464 - 0.755}{8.01 - 0.755} = 0.9247$

and $h_8 = 226 + 0.9247 \times (2598 - 226) = 2419 \text{ kJ/kg}$

Work done, $w_{78} = h_7 - h_8 = 3264 - 2419 = 845 \text{ kJ/kg}$

Thus total work output, $\text{Net} = w_{5G} + w_{78} + w_{12}$

$= 196 + 845 - 2 = 1039 \text{ kJ/kg}$

Total energy added, $q_{in} = (h_8 - h_2) + (h_7 - h_6)$

$= \left(3248 - 228\right) + \left(3264 - 3052\right)$

$= 3232 \text{ kJ/kg}$

Thus thermal efficiency,

$\eta_{th} = \frac{\text{Net}}{q_{in}} = \frac{1039}{3232} = 32.14\%$

Reheat has improved the dryness of the steam and the thermal efficiency.