Contents

Solutions to Chapter 2

Exercises

2.1 Simplify the following Boolean functions

a) $\overline{\overline{(A \cap B \cup C)} \cap \bar{B}}$

$= (A \cap B \cup C) \cup B$

$= AB \cup B \cup C$

$= \boldsymbol{B} \cup \boldsymbol{C}$

b) $(A \cup B) \cap (\bar{A} \cup \bar{B} \cap \bar{A})$

$= (A \cup B) \cap (\bar{A})$

$= (A\bar{A} \cup B\bar{A})$

$= \boldsymbol{B} \cap \bar{\boldsymbol{A}}$

c) $\overline{A \cap B \cap B \cap C} \cap \bar{B}$

$= \overline{(A \cap B \cap B \cap C)} \cap \bar{B}$

$= (\bar{A} \cup \bar{B} \cup \bar{B} \cup \bar{C}) \cap \bar{B}$

$= (\bar{A} \cup \bar{B} \cup \bar{C}) \cap \bar{B}$

$= \bar{A} \cap \bar{B} \cup \bar{B} \cap \bar{B} \cup \bar{C} \cap \bar{B}$

$= \bar{A}\bar{B} \cup \bar{B}\bar{B} \cup \bar{C}\bar{B}$

$= \bar{A}\bar{B} \cup \bar{B} \cup \bar{C}\bar{B}$

$= \bar{\boldsymbol{B}}$

2.2 Reduce the following Boolean function

$A \cap B \cap \left(\overline{C \cup (\bar{C} \cup A) \cup \bar{B}}\right)$

$= A \cap B \cap \left(\bar{C} \cap \left(\overline{\bar{C} \cup A}\right) \cap B\right)$

$= A \cap B \cap (\bar{C} \cap (C \cap \bar{A}) \cap B)$

$= AB\bar{C}C\bar{A}B$

$= \phi$

2.3 Simplify the following Boolean expressions

a) $\overline{\overline{(A \cap B \cup C)} \cap \bar{B}}$

$= (A \cap B \cup C) \cup B$

$= AB \cup B \cup C$

$= \boldsymbol{B} \cup \boldsymbol{C}$

b) $[(A \cup B) \cap \bar{A}] \cup (\bar{B} \cap \bar{A})$

$= [A \cap \bar{A} \cup B \cap \bar{A}] \cup (\bar{B} \cap \bar{A})$

$= [\phi \cup B \cap \bar{A}] \cup (\bar{B} \cap \bar{A})$

$= B\bar{A} \cup \bar{B}\bar{A}$

$= \bar{A}(B \cup \bar{B})$

$= \boldsymbol{\bar{A}}$

2.4 Reduce the following Boolean function

$G = (A \cup B \cup C) \cap \left(\overline{A \cap \bar{B} \cap \bar{C}}\right) \cap \bar{C}$

$G = (A \cup B \cup C) \cap (\bar{A} \cup B \cup C) \cap \bar{C}$

$G = (A \cup B \cup C) \cap (\bar{A}\bar{C} \cup B\bar{C} \cup C\bar{C})$

$G = (A \cup B \cup C) \cap (\bar{A}\bar{C} \cup B\bar{C})$

$G = A\bar{A}\bar{C} \cup B\bar{A}\bar{C} \cup C\bar{A}\bar{C} \cup AB\bar{C} \cup BB\bar{C} \cup CB\bar{C}$

$G = \phi \cup B\bar{A}\bar{C} \cup \phi \cup AB\bar{C} \cup B\bar{C} \cup \phi$

$\boldsymbol{G = B\bar{C}}$

If Pr(A) = Pr(B) = Pr(C) = 0.9, what is Pr(G)?

$\mathbf{Pr}(\boldsymbol{G}) = \mathbf{0.9} \times \mathbf{0.1} = \mathbf{0.09}$

2.5 Simplify the following Boolean equations

a) $(A \cup B \cup C) \cap \left(\overline{A \cap \bar{B} \cap \bar{C}}\right) \cap \bar{C}$

$= (A \cup B \cup C) \cap (\bar{A} \cup B \cup C) \cap \bar{C}$

$= (A \cup B \cup C) \cap (\bar{A}\bar{C} \cup B\bar{C} \cup C\bar{C})$

$= (A \cup B \cup C) \cap (\bar{A}\bar{C} \cup B\bar{C})$

$= A\bar{A}\bar{C} \cup B\bar{A}\bar{C} \cup C\bar{A}\bar{C} \cup AB\bar{C} \cup BB\bar{C} \cup CB\bar{C}$

$= \phi \cup B\bar{A}\bar{C} \cup \phi \cup AB\bar{C} \cup B\bar{C} \cup \phi$

$= \boldsymbol{B\bar{C}}$

b) $(A \cup B) \cap \bar{B}$

$= A\bar{B} \cup B\bar{B}$

$= \boldsymbol{A\bar{B}}$

2.6 Reduce the following Boolean equation

$\left(\overline{\mathbf{A} \cup (\mathbf{B} \cap \mathbf{C})}\right) \cap \left(\overline{\mathbf{B} \cup (\mathbf{D} \cap \mathbf{A})}\right)$

$= \left(\bar{A} \cap (\bar{B} \cup \bar{C})\right) \cap \left(\bar{B} \cap (\bar{D} \cup \bar{A})\right)$

$= (\bar{A}\bar{B} \cup \bar{A}\bar{C}) \cap (\bar{B}\bar{D} \cup \bar{B}\bar{A})$

$= \bar{A}\bar{B}\bar{B}\bar{D} \cup \bar{A}\bar{C}\bar{B}\bar{D} \cup \bar{A}\bar{B}\bar{B}\bar{A} \cup \bar{A}\bar{C}\bar{B}\bar{A}$

$= \bar{A}\bar{B}\bar{D} \cup \bar{A}\bar{C}\bar{B}\bar{D} \cup \bar{A}\bar{B} \cup \bar{A}\bar{C}\bar{B}\bar{A}$

$= \boldsymbol{\bar{A}\bar{B}}$

2.7 Use both Equations (2.25) and (2.29) to find the reliability Pr(s). Which equation is preferred for numerical solution?

$\Pr(s) = \Pr(E_1 \cup E_2 \cup E_3)$

$\Pr(E_1) = 0.8; \quad \Pr(E_2) = 0.9; \quad \Pr(E_3) = 0.95$

Using Equation 2.25:

$\Pr(E_1 \cup E_2 \cup E_3)$

$= [\Pr(E_1) + \Pr(E_2) + \Pr(E_3)] - [\Pr(E_1 \cap E_2) + \Pr(E_1 \cap E_3) + \Pr(E_2 \cap E_3)] + \Pr(E_1 \cap E_2 \cap E_3)$

$= [0.8 + 0.9 + 0.95] - [0.8 \times 0.9 + 0.8 \times 0.95 + 0.9 \times 0.95] + 0.8 \times 0.9 \times 0.95$

$= \mathbf{0.999}$

Using Equation 2.29 (Preferred Equation for Computational Simplicity)

$\Pr(E_1 \cup E_2 \cup E_3)$

$= 1 - [1 - \Pr(E_1)][1 - \Pr(E_2)][1 - \Pr(E_3)]$

$= 1 - [1 - 0.8][1 - 0.9][1 - 0.95]$

$= 1 - 0.001$

$= \mathbf{0.999}$

2.8 A stockpile of 40 relays contain 8 defective relays. If five relays are selected at random and the number of defective relays is known to be greater than two, what is the probability that exactly four relays are defective?

x = number of defective relays in selected sample = 4
n = number of relays in the selected sample = 5
N = number of relays in the stockpile = 40
D = number of defective relays in the stockpile = 8

$$\Pr(X = 4\,|X > 2) = \frac{\Pr(X > 2\,|X = 4) \times \Pr(X = 4)}{\Pr(X > 2)} = \frac{\Pr(X = 4)}{\Pr(X > 2)}$$

This problem can be solved using the **Hypergeometric Distribution**

$$\Pr(X = x) = \frac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}}$$

$$\Pr(X = 4) = \frac{\binom{8}{4}\binom{40-8}{5-4}}{\binom{40}{5}} = 0.003404$$

Similarly:
$\Pr(X = 3) = 0.0422$
$\Pr(X = 5) = 8.51 \times 10^{-5}$

$$\Pr(X = 4\,|X > 2) = \frac{0.003404}{0.0422 + 0.003404 + 8.51 \times 10^{-5}} = \mathbf{0.074488}$$

2.9 Given that P = 0.006 is the probability of an engine failure on a flight between two cities, find the probability of:

a) No engine failure in 1000 flights
b) At least one failure in 1000 flights
c) At least two failures in 1000 flights

Method 1: Using the Binomial Distribution

Given n =1000; p = 0.006

Let X be the r.v. of having an engine failure during a flight

$$\Pr(X = x) = \binom{1000}{x} 0.006^x \times 0.994^{1000-x}$$

a) $\Pr(X = 0) = \binom{1000}{0} 0.006^0 \times 0.994^{1000} = \mathbf{0.00243}$

b) $\Pr(X \geq 1) = 1 - \binom{1000}{0} 0.006^0 \times 0.994^{1000} = \mathbf{0.99757}$

c) $\Pr(X \geq 2) = 1 - \binom{1000}{1} 0.006^1 \times 0.994^{999} - \binom{1000}{0} 0.006^0 \times 0.994^{1000} = \mathbf{0.98287}$

Method 2: Using the Poisson Distribution

If n >> 1 & p << 1, the Poisson distribution can be used to approximate the Binomial distribution.

$$\Pr(X = x) = \frac{\mu^x e^{-\mu}}{x!} \qquad where\ \mu = np = 1000 \times 0.006 = 6$$

a) $\Pr(X = 0) = \frac{6^0 e^{-6}}{0!} = \mathbf{0.00248}$

b) $\Pr(X \geq 1) = 1 - \frac{6^0 e^{-6}}{0!} = \mathbf{0.99752}$

c) $\Pr(X \geq 2) = 1 - \frac{6^1 e^{-6}}{1!} - \frac{6^0 e^{-6}}{0!} = \mathbf{0.98265}$

2.10 A random sample of 10 resistors is to be tested. From past experience, it is known that the probability of a given resistor being defective is 0.08. Let X be the r.v. for number of defective resistors.

a) What kind of distribution function would be recommended for modeling the r.v.?

This problem is best modeled using the Binomial distribution

b) According to the distribution function in (a), what is the probability that in the sample of 10 resistors, there are more than 1 defective resistors in the sample?

Given n =10; p = 0.08

Let X be the r.v. of being given a defective resistor

$$\Pr(X > 1) = 1 - \binom{10}{1} 0.08^1 \times 0.92^9 - \binom{10}{0} 0.08^0 \times 0.92^{10} = \mathbf{0.1879}$$

2.11 How many different license plates can be made if each consists of three numbers and three letters, and no number or letter can appear more than once on a single plate?

This problem is a question of determining the number of possible permutations. Assuming that the digits 0 through to 9 can be used and that all 26 letters of the alphabet can be used, then the solution is given by:

$$Let\ L\ = the\ r.v. for\ the\ letters$$

$$Let\ N\ = the\ r.v. for\ the\ numbers$$

$$No. of\ Different\ Plates\ = L \times L \times L \times N \times N \times N$$

$$No. of\ Different\ Plates\ = 26 \times 25 \times 24 \times 10 \times 9 \times 8 = \mathbf{11,232,000}$$

2.12 The consumption of maneuvering jet fuel in a satellite is known to be normally distributed with a mean of 10,000 hours and a standard deviation of 1,000 hours. What is the probability of being able to maneuver the satellite for the duration of a 1-year mission?

Let T be the r.v. for the consumption life of the satellite jet fuel

$$\Pr(T = t) \sim Norm(\mu = 10{,}000; \sigma = 1{,}000)$$

$$T = 1yr = 8760\ hrs$$

$$\Pr(T > 8760) = \Pr\left(Z > \frac{T - \mu}{\sigma}\right) = \Pr\left(Z > \frac{8760 - 10000}{1000}\right)$$

$$\Pr(Z > -1.24) = \mathbf{0.8925}$$

2.13 Suppose a process produces electronic components, 20% of which are defective. Find the distribution of x, the number of defective components in a sample size of five. Given that the sample contains at least three defective components, find the probability that four components are defective.

This is a conditional probability with the Binomial distribution with parameters N = 5 and p = 0.2

$$\Pr(X = 4\,|X \geq 3) = \frac{\Pr(X \geq 3\,|X = 4) \times \Pr(X = 4)}{\Pr(X \geq 3)} = \frac{\Pr(X = 4)}{\Pr(X \geq 3)}$$

$$\Pr(X = 4\,|X \geq 3) = \frac{\binom{5}{4} 0.2^4 \times 0.8^1}{\binom{5}{3} 0.2^3 \times 0.8^2 + \binom{5}{4} 0.2^4 \times 0.8^1 + \binom{5}{5} 0.2^5 \times 0.8^0}$$

$$\Pr(X = 4\,|X \geq 3) = \frac{0.0064}{0.0512 + 0.0064 + 0.00032} = \mathbf{0.1105}$$

2.14 If the heights of 300 students are normally distributed, with a mean of 68 inches and standard deviation of 3 inches, how many students have:

a) heights of more than 70 inches?

$$\Pr(X > 70) = \Pr\left(Z > \frac{X-\mu}{\sigma}\right) = \Pr\left(Z > \frac{70-68}{3}\right) = \Pr\left(Z > \frac{2}{3}\right) = \mathbf{0.2525}$$

Therefore, the number of students that are greater than 70 inches in height is given by:

$$300 \times 0.2525 = \mathbf{76}$$

b) heights between 67 and 68 inches?

$$\Pr(67 < X > 68) = \Pr\left(\frac{67-68}{3} < Z < \frac{68-68}{3}\right) = \Pr\left(\frac{-1}{3} < Z < 0\right) = \mathbf{0.13056}$$

Therefore, the number of students with heights between 67 and 68 inches is given by:

$$300 \times 0.13056 = \mathbf{39}$$

2.15 Assume that for a certain type of resistor, 1% are bad when purchased. What is the probability that a circuit with 10 resistors has exactly 1 bad resistor?

Let X be the r.v. of finding a bad resistor

This is a binomial distribution with parameters N = 10 and p = 0.01

$$\Pr(X = 1) = \binom{10}{1} 0.01^1 \times 0.99^9 = \mathbf{0.0914}$$

2.16 Between the hours of 2 and 4 p.m. the average number of phone calls per minute coming into an office is two and one-half. Find the probability that during a particular minute, there will be more than five phone calls.

This is a Poisson Distribution

$$\Pr(X = x) = \frac{\mu^x e^{-\mu}}{x!} \qquad where\ \mu = \lambda t$$

$$Given\ \lambda = 2.5\ min^{-1}\ \&\ t = 1\ min;\ \mu = 2.5$$

$$\Pr(X > 5) = 1 - \sum_{i=1}^{5} \frac{\mu^x e^{-\mu}}{x!}$$

$$\Pr(X > 5) = 1 - 0.95798 = \mathbf{0.04202}$$

2.17 A guard works between 5 p.m. and 12 midnight; he sleeps an average of 1 hour before 9 p.m., and 1.5 hours between 9 and 12. An inspector finds him asleep, what is the probability that this happens before 9 p.m.?

Let A_1 be the fraction of time between 5-9pm

Let A_2 be the fraction of time between 9-12pm

Let E be the event of the guard sleeping on shift

$\Pr(A_1) = {}^4/_7 \quad \Pr(A_2) = {}^3/_7$

$\Pr(E \mid A_1) = {}^1/_4 \quad \Pr(E \mid A_2) = {}^{1.5}/_3$

Using Bayes Theorem

$$Pr(A_j|E) = \frac{Pr(E|A_j)\Pr(A_j)}{\sum_{j=1}^{n} Pr(E|A_j)\Pr(A_j)}$$

$$Pr(A_1|E) = \frac{Pr(E|A_1)\Pr(A_1)}{Pr(E|A_1)\Pr(A_1) + Pr(E|A_2)\Pr(A_2)} = \frac{{}^1/_4 \times {}^4/_7}{{}^1/_4 \times {}^4/_7 + {}^{1.5}/_3 \times {}^3/_7} = \mathbf{0.4}$$

2.18 The number of system breakdowns occurring with a constant rate in a given length of time has a mean value of two breakdowns. What is the probability that in the same length of time, two breakdowns will occur?

This is a Poisson Distribution

$$\Pr(X = x) = \frac{\mu^x e^{-\mu}}{x!} \qquad where\ \mu = \lambda t$$

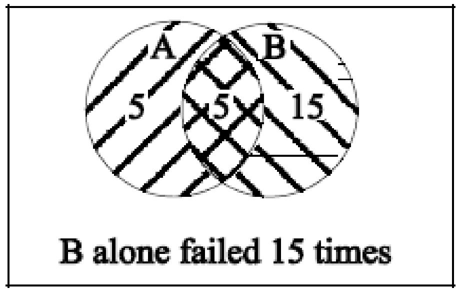
$Given\ \ \mu = 2$

$$\Pr(X = 2) = \frac{2^2 e^{-2}}{2!} = \mathbf{0.271}$$

2.19 An electronic assembly consists of two subsystems, A and B. Each assembly is given one preliminary checkout test. Records on 100 preliminary checkout tests show that subsystem A failed 10 times. Subsystem B alone failed 15 times. Both subsystems A and B failed together five times.

a) What is the probability of A failing, given that B has failed.

b) What is the probability that A alone fails.



Total failures of subsystem A = 10
Simultaneous failures of subsystems A & B = 5
Total failures of subsystem B = 15+5

$\Pr(B) = 0.2$
$\Pr(A \cap B) = 0.05$

Using Law of Conditional Probability:

$$a)\ Pr(A|B) = \frac{Pr(A \cap B)}{\Pr(B)} = \frac{0.05}{0.2} = \mathbf{0.25}$$

Using Law of Total Probability:

$$b)\ Pr(A\ alone) = \sum_{i=1}^{n} Pr(A|B_i)\Pr(B_i) = 0.25 \times 0.2 = \mathbf{0.05}$$

2.20 A presidential election poll shows one candidate leading with 60% of the vote. If the poll is taken from 200 random voters throughout the U.S., what is the probability that the candidate will get less than 50% of the votes in the election? (Assume the 200 voters sampled are true representatives of the voting profile.)

Using a Binomial approximation for a Normal Distribution we find the solution as follows:

Given n = 200 & p = 0.6
$\mu = np = 200 \times 0.6 = 120$
$\sigma^2 = npq = 200 \times 0.6 \times 0.4 = 48$

$$Pr(X < 100) = \Pr\left(Z < \frac{100 - 120}{\sqrt{48}}\right) = \Pr(Z < -2.887) = \mathbf{0.001946}$$

2.21 A newspaper article reports that a New York medical team has introduced a new male contraceptive method. The effectiveness of this method was tested using a number of couples over a period of 5 years. The following statistics are obtained:

Year	Times Employed	Unwanted Pregnancies (X)
1	8200	19
2	10100	18
3	2120	1
4	6120	9
5	18130	30

a) Estimate the mean probability of an unwanted pregnancy per use. What is the standard deviation of the estimate?