4.1

Since it leads to an important result, let us first do it for the general case where X ~ N(λ, ζ), i.e.

$$f_X(x) = \frac{1}{\sqrt{2\pi}\zeta} e^{-\frac{1}{2}\left(\frac{x-\lambda}{\zeta}\right)^2}$$

To obtain PDF $f_Y(y)$ for $Y = e^X$, one may apply Ang & Tang (4.6),

$$f_Y(y) = f_X(g^{-1}) \left|\frac{dg^{-1}}{dy}\right|$$

where

$$y = g(x) = e^x;$$

$\Rightarrow x = g^{-1}(y) = \ln(y)$

$\Rightarrow \left|\frac{dg^{-1}}{dy}\right| = \left|\frac{d}{dy}\ln y\right| = \left|\frac{1}{y}\right|$

$= \frac{1}{y}$ since $y = e^X > 0$

Hence, expressed as a function of y, the PDF $f_Y(y)$ is

$f_X(g^{-1}) \left|\frac{dg^{-1}}{dy}\right| = f_X(\ln(y)) \frac{1}{y}$

$= \frac{1}{\sqrt{2\pi}y\zeta} e^{-\frac{1}{2}\left(\frac{\ln y-\lambda}{\zeta}\right)^2}$ (non-negative y only)

which, by comparison to Ang & Tang (3.29), is exactly what is called a log-normal distribution with parameters λ and ζ, i.e. if X ~ N(λ, ζ), and $Y = e^X$, then Y ~ LN(λ, ζ).

Hence, for the particular case where X ~ N(2, 0.4), Y is LN with parameters λ being 2 and ζ being 0.4.

4.2

To have a better physical feel in terms of probability (rather than probability density), let's work with the CDF (which we can later differentiate to get the PDF) of Y: since Y cannot be negative, we know that $P(Y < 0) = 0$, hence

when $y < 0$:

$$F_Y(y) = 0$$
$$\Rightarrow f_Y(y) = [F_Y(y)]' = 0$$

But when $y \geq 0$,

$$F_Y(y) = P(Y \leq y)$$

$$= P(\frac{1}{2} m X^2 \leq y)$$

$$= P(-\sqrt{\frac{2y}{m}} \leq X \leq \sqrt{\frac{2y}{m}})$$

$$= F_X\left(\sqrt{\frac{2y}{m}}\right) - F_X\left(-\sqrt{\frac{2y}{m}}\right)$$

$$= F_X\left(\sqrt{\frac{2y}{m}}\right) - 0$$

$$= F_X\left(\sqrt{\frac{2y}{m}}\right)$$

Hence the PDF,

$$f_Y(y) = \frac{d}{dy}[F_Y(y)]$$

$$= f_X\left(\sqrt{\frac{2y}{m}}\right)\frac{d}{dy}\sqrt{\frac{2y}{m}}$$

$$= \frac{8y}{ma^3\sqrt{\pi}}\exp(-\frac{2y}{ma^2})\ \frac{1}{\sqrt{2my}}$$

$$= \frac{4}{a^3}\sqrt{\frac{2y}{\pi m^3}}\exp(-\frac{2y}{ma^2})$$

Hence the answer is

$$f_Y(y) = \begin{cases} \frac{4}{a^3}\sqrt{\frac{2y}{\pi m^3}}\exp(-\frac{2y}{ma^2}) & y \geq 0 \\ 0 & y < 0 \end{cases}$$

4.3

A = volume of air traffic
C = event of overcrowded

(a) T = total power supply = $N(\mu_T, \sigma_T)$

Where $\mu_T = 100 + 200 + 400$

$\sigma_T = \sqrt{15^2 + 40^2 + 40^2} = 58.5$

(b) P(Normal weather) = P(W) = 2/3
P(Extreme weather) = P(E) = 1/3
P(Power shortage) = P(S) = P(S|W)P(W)+P(S|E)P(E)
= P(T<400)x2/3 + P(T<600)x1/3

$= [\Phi(\frac{400-400}{58.5})]\times\frac{2}{3} + [\Phi(\frac{600-400}{58.5})]\times\frac{1}{3}$

$= [\Phi(0)]\times\frac{2}{3} + [\Phi(3.42)]\times\frac{1}{3}$

= 0.5x0.667 + 0.99968x0.333
= 0.667

(c) $P(W|S) = \frac{P(S|W)P(W)}{P(S)} = \frac{0.5\times 0.667}{0.667} = 0.5$

(d) P(all individual power source can meet respective demand)
= P(N>0.15x400)P(F>0.3x400)P(H>0.55x400)

$= [1-\Phi(\frac{60-100}{15})][1-\Phi(\frac{120-200}{40})][1-\Phi(\frac{220-400}{40})]$

= Φ (2.67)xΦ (2)xΦ (4.5)
= 0.09962x0.977x1
= 0.973

P(at least one source not able to supply respective allocation) = 0.027

4.4

(a) Let T_J be John's travel time in (hours); $T_J = T_3 + T_4$ with

$$\mu_{T_J} = 5 + 4 = 9 \text{ (hours), and}$$

$$\sigma_{T_J} = [3^2 + 1^2 + (2)(0.8)(3)(1)]^{1/2} = 3.847 \text{ (hours)}$$

Hence

$$P(T_J > 10 \text{ hours}) = 1 - P(\frac{T_J - \mu_{T_J}}{\sigma_{T_J}} \leq \frac{10-9}{3.847})$$

$$= 1 - \Phi(0.26) = 1 - 0.603$$

$$\cong \mathbf{0.397}$$

(b) Let T_B be Bob's travel time in (hours); $T_B = T_1 + T_2$ with

$$\mu_{T_B} = 6 + 4 = 10 \text{ (hours), and}$$

$$\sigma_{T_B} = [2^2 + 1^2]^{1/2} = \sqrt{5} \text{ (hours)}$$

Hence

$$P(T_J - T_B > 1) = P(T_B - T_J + 1 < 0),$$

now let $R \equiv T_B - T_J + 1$; R is normal with

$$\mu_R = \mu_{T_B} - \mu_{T_J} + 1 = 10 - 9 + 1 = 2,$$

$$\sigma_R = [\sigma_{T_B}{}^2 + \sigma_{T_J}{}^2]^{1/2} = (5 + 14.8)^{1/2} = \sqrt{19.8},$$

hence

$$P(R < 0) = \Phi(\frac{0-2}{\sqrt{19.8}})$$

$$= \Phi(-0.449)$$

$$\cong \mathbf{0.327}$$

(c) Since the lower route (A-C-D) has a smaller expected travel time of μ_{T_J} = 9 hours as compared to the upper (with expected travel time = μ_{T_B} = 10 hours), one should take the **lower** route to minimize expected travel time from A to D.

4.5

(a) To calculate probability, we first need to have the PDF of *S*. As a linear combination of three normal variables, *S* itself is normal, with parameters

$$\mu_S = 0.3\times5 + 0.2\times8 + 0.1\times7 = 3.8 \text{ (cm)}$$
$$\sigma_S = \sqrt{0.3^2 1^2 + 0.2^2 2^2 + 0.1^2 1^2} \cong 0.51 \text{ (cm)}$$

Hence

$$P(S > 4\text{cm}) = 1 - \Phi\left(\frac{4-3.8}{0.51}\right)$$
$$= 1 - \Phi(0.3922) = 1 - 0.6526$$
$$\cong \mathbf{0.347}$$

(b) Now that we have a constraint $A + B + C = 20$m, these variables are no longer all independent, for example, we have

$$C = 20 - A - B$$

Hence

$$S = 0.3A + 0.2B + 0.1(20 - A - B)$$
$$\Rightarrow S = 0.2A + 0.1B + 2, \text{ with } \rho_{AB} = 0.5.$$

Thus

$$\mu_S = 0.2\times5 + 0.1\times8 + 2 = 3.8 \text{ (cm) as before, and}$$

$$\sigma_S = \sqrt{0.2^2 1^2 + 0.1^2 2^2 + 2\times0.5\times0.1\times1\times2}$$
$$= \sqrt{0.12} \cong 0.346 \text{ (cm)}$$

Hence

$$P(S > 4\text{cm}) = 1 - \Phi\left(\frac{4-3.8}{\sqrt{0.12}}\right)$$
$$= 1 - \Phi(0.577) \cong \mathbf{0.282}$$

4.6

$$Q = 4A + B + 2C$$
$$= 4A + B + 2(30 - A - B)$$
$$= 2A - B + 60, \text{ hence}$$
$$\mu_Q = 2\times5 - 8 + 60 = 62,$$
$$\sigma_Q = [4\times3^2 + 1\times2^2 + 2(2)(-1)(-0.5)(3)(2)]^{1/2}$$
$$= (36 + 4 + 12)^{1/2} = \sqrt{52}$$

$$\text{Hence } P(Q < 40) = P\left(\frac{Q - \mu_Q}{\sigma_Q} < \frac{40 - 62}{\sqrt{52}}\right)$$
$$= \Phi(-3.0508)$$
$$\cong \mathbf{0.00114}$$

4.7

(a) Let Q_1 and Q_2 be the annual maximum flood peak in rivers 1 and 2, respectively. We have

$$Q_1 \sim N(35, 10),\ Q_2 \sim N(25, 10)$$

The annual max. peak discharge passing through the city, Q, is the sum of them,

$$Q = Q_1 + Q_2, \text{ hence}$$

$$\mu_Q = \mu_{Q_1} + \mu_{Q_2} = 35 + 25 = \mathbf{60} \text{ (m}^3\text{/sec), and}$$

$$\sigma_Q^2 = \sigma_{Q_1}^2 + \sigma_{Q_2}^2 + 2\,\rho_{Q_1Q_2}\sigma_{Q_1}\sigma_{Q_2}$$

$$= 10^2 + 10^2 + 2\times0.5\times10\times10 = 300 \text{ (m}^3\text{/sec),}$$

$$\Rightarrow \sigma_Q = \sqrt{300} \cong \mathbf{17.32} \text{ (m}^3\text{/sec)}$$

(b) The annual risk of flooding, $p = P(Q > 100)$

$$= 1 - \Phi\left(\frac{100-60}{\sqrt{300}}\right)$$

$$= 1 - \Phi(2.309) = 1 - 0.9895$$

$$= \mathbf{0.0105} \text{ (probability each year)}$$

Hence the return period is $\tau = \frac{1}{p}$

$$= \frac{1}{0.0105} = 95.59643882$$

$$\cong \mathbf{95\ years}.$$

(c) Since the yearly risk of flooding is $p = 0.0105$, and we have a course of $n = 10$ years, we adopt a binomial model for X, the total number of flood years over a 10-year period.

P(city experiences (any) flooding) = 1 – P(city experiences no flooding at all)

$$= 1 - P(X = 0)$$

$$= 1 - (1-p)^{10} = 1 - (1 - 0.0105)^{10}$$

$$= 1 - 0.9895^{10}$$

$$\cong \mathbf{10\%}$$

(d) The requirement on p is, using the flooding probability expression from part (c):

$$1 - (1-p)^{10} = 0.1 \div 2 = 0.05$$

$$\Rightarrow p = 1 - (1 - 0.05)^{1/10} = 0.0051,$$

which translates into a condition on the design channel capacity Q_0, following what's done in (b),

$$1 - \Phi\left(\frac{Q_0 - 60}{\sqrt{300}}\right) = 0.0051$$

$$\Rightarrow \Phi\left(\frac{Q_0 - 60}{\sqrt{300}}\right) = 0.9949$$

$$\Rightarrow Q_0 = 60 + \Phi^{-1}(0.9949)\ \sqrt{300}$$

$$= 60 + 2.57\sqrt{300} = 104.5$$

$\therefore$ Extending the channel capacity to about **104.5 m^3/sec** will cut the risk by half.

4.8

$$P(T<30)=P(T<30|N=0)P(N=0)+P(T<30|N=1)P(N=1)+P(T<30|N=2)P(N=2)$$

$$=\Phi(\frac{30-30}{5})\times 0.2+\Phi(\frac{30-35}{\sqrt{25+3^2}})\times 0.5+\Phi(\frac{30-40}{\sqrt{25+2\times 3^2}})\times 0.3$$

$$=0.5\times 0.2+\Phi(-0.857)\times 0.5+\Phi(-1.525)\times 0.3$$

$$=0.217$$

4.9

N = total time for one round trip in normal traffic

= N(30+20+40, $\sqrt{9^2+4^2+12^2}$)

= N(90, 15.52)

R = total round trip time in rush hour traffic

= N(30+30+40, $\sqrt{9^2+6^2+12^2}$)

= N(100, 16.16)

(a) P(on schedule in normal traffic)

= P(N<20)

$= \Phi(\frac{120-90}{15.52}) = \Phi(1.933) = 0.974$

(b) T_A = Normal time taken for passenger starting from A = $T_2 + T_3$

= N(20+40, $\sqrt{4^2+12^2}$) = N(60, 12.65)

$P(T_A < 60) = \Phi(\frac{60-60}{12.65}) = \Phi(0) = 0.5$

(c) Under rush hour traffic

T_A = N(30+40, $\sqrt{6^2+12^2}$) = N(70, 13.4)

T_B = N(40, 12)

$P(T_A<60) = \Phi(\frac{60-70}{13.4}) = \Phi(-0.746) = 0.227$

$P(T_B<60) = \Phi(\frac{60-40}{12}) = \Phi(1.667) = 0.952$

Percentage of passengers arriving in less than an hour

= 0.227x1/3 + 0.952x2/3

= 0.7

(d) $P(T_B < 60 \mid T_B > 45) = \frac{P(45 < T_B < 60)}{P(T_B > 45)}$

$$= \frac{\Phi(\frac{60-40}{12}) - \Phi(\frac{45-40}{12})}{1-\Phi(\frac{45-40}{12})} = \frac{\Phi(1.667)-\Phi(0.417)}{1-\Phi(0.417)} = \frac{0.952-0.661}{0.339} = 0.858$$

4.10

(a) $D=S_1-S_2$

$$\mu_D = \mu_{S_1} - \mu_{S_2} = 2-2=0$$

$$Var(D) = Var(S_1) + Var(S_2) - 2\rho\sqrt{Var(S_1)Var(S_2)}$$

$$= (0.3\times 2)^2 + (0.3\times 2)^2 - 2\times 0.7(0.3\times 2)^2$$

$$= .216$$

(b) $P(-.5 < D < .5) = \Phi(\frac{0.5-0}{\sqrt{0.216}}) - \Phi(\frac{-0.5-0}{\sqrt{0.216}}) = \Phi(1.076) - \Phi(-1.076) = 0.718$

4.11

(a) $X_5 = X_1 + X_2 + X_3$

$\mu_5 = \mu_1 + \mu_2 + \mu_3 = 10 + 15 + 20 = 45$

$\sigma_5^2 = \sigma_1^2 + \sigma_2^2 + 2\rho_{1,2}\sigma_1\sigma_2 = 3^2 + 3^2 + 2 \times 0.6 \times 3 \times 3 = 28.8$

$\therefore \sigma_5 = 5.37$

(b) $V_1 = 60X_1$

$V_2 = 60X_2$

$\therefore P(V_2 - V_1 > 400) = P(Z > 400)$

$\therefore \mu_Z = 60\mu_2 - 60\mu_1 = 60 \times 15 - 60 \times 10 = 300$

$\sigma_Z^2 = 60^2\sigma_2^2 + 60^2\sigma_1^2 - 2\rho_{1,2} \times 60^2\sigma_1\sigma_2 = 25920$

$\therefore \sigma_Z = 161$

$\therefore P(V_2 - V_1 > 400) = 1 - \Phi(\frac{400-300}{161}) = 1 - \Phi(.621) = .267$

(c) $X_5 = X_1 + X_2 + X_3$

$\mu_5 = \mu_1 + \mu_2 + \mu_3 = 10 + 15 + 20 + 3n = 45 + 3n$

$\sigma_5^2 = \sigma_1^2 + \sigma_2^2 + 2\rho_{1,2}\sigma_1\sigma_2 = 3^2 + 3^2 + 2 \times 0.6 \times 3 \times 3 = 28.8$

$\therefore \sigma_5 = 5.37$

$\because P(X_5 > 70) = .05$

$\therefore P(X_5 < 70) = .95$

$$\Phi\left(\frac{70 - 45 - 3n}{5.37}\right) = .95$$

$\Rightarrow 25 - 3n = 5.37\Phi^{-1}(.95) = 5.37 * 1.645 = 8.839$

$\Rightarrow n = 5.4$

4.12

(a) P(X>20∪Y>20)=P(X>20)+P(Y>20)-P(X>20)P(Y>20)

$$= (1-\Phi(\frac{20-20}{4})) + (1-\Phi(\frac{20-15}{3})) - (1-\Phi(\frac{20-20}{4})) \times (1-\Phi(\frac{20-15}{3}))$$

=0.5+0.0478-0.5×0.0478

=0.524

(b) Z in normal distribution with

$\mu_Z = 0.6 \times 20 + 0.4 \times 15 = 18$

$\sigma_Z = \sqrt{0.6^2 4^2 + 0.4^2 3^2} = 2.683$

$\therefore P(Z > 20) = 1 - \Phi(\frac{20-18}{2.683}) = 1 - 0.7718 = 0.2282$

(c) $\mu_Z = 0.6 \times 20 + 0.4 \times 15 = 18$

$\sigma_Z = \sqrt{0.6^2 4^2 + 0.4^2 3^2 + 2 \times 0.8 \times 0.6 \times 0.4 \times 4 \times 3} = 3.436$

$\therefore P(Z > 20) = 1 - \Phi(\frac{20-18}{3.436}) = 1 - 0.72 = 0.28$

4.13

(a) $P(X \geq 2) = 1 - P(X=0) - P(X=1)$

$$= 1 - \binom{5}{0} 0.6^0 0.4^5 - \binom{5}{1} 0.6^1 0.4^4$$

$$= 1 - 0.4^5 - 5 \times 0.6 \times 0.4^4$$

$$= 0.046$$

(b) N_H, N_B are the number of highway and building jobs won

$P(N_H = 1 \cap N_B = 0) = P(N_H = 1) P(N_B = 0)$

$$= \binom{3}{1} 0.6^1 0.4^2 \binom{2}{0} 0.6^0 0.4^2$$

$$= 0.046$$

(c) $T = H_1 + H_2 + B$ where H_1, H_2 and B are the profits from the respective jobs.

T in Normal distribution with

$$\mu_T = 100 + 100 + 80 = 280$$

$$\sigma_T = \sqrt{40^2 + 40^2 + 20^2} = 60$$

$$\therefore P(T > 300) = 1 - \Phi(\frac{300 - 280}{60}) = 1 - \Phi(0.333) = 0.2695$$

(d) $$\mu_T = 100 + 100 + 80 = 280$$

$$\sigma_T = \sqrt{40^2 + 40^2 + 20^2 + 2 \times 0.8 \times 40 \times 40} = 78.5$$

$$\therefore P(T > 300) = 1 - \Phi(\frac{300 - 280}{78.5}) = 1 - \Phi(0.255) = 0.4$$

4.14

S: the amount of water available. S follows LogNormal Distribution with

E(S)=1, c.o.v=0.4

$$\zeta_S^2 = \ln(1+0.4^2) = 0.14842$$

$$\lambda_S = \ln\mu_S - \tfrac{1}{2}\zeta_S^2 = \ln 1 - 0.5\times 0.14842 = -0.07421$$

D: the total demand of water.

E(D)=1.5, c.o.v=0.1

$$\zeta_D^2 = 0.1^2 = 0.01$$

$$\lambda_D = \ln\mu_D - \tfrac{1}{2}\zeta_D^2 = \ln 1.5 - 0.5\times 0.01 = 0.4005$$

P(water shortage)=P(D>S)

=P(D/S>1)

=P((Z=ln(D/S))>0)

Z follows Normal Distribution with:

$$Var(Z) = \zeta^2{}_D + \zeta_S^2 = 0.1^2 + 0.148 = 0.158$$

$$\mu_Z = \lambda_D - \lambda_S = 0.4005 - (-0.07421) = 0.4747$$

$$\Rightarrow P(Z>0) = 1 - \Phi\left(\frac{0-0.4747}{\sqrt{0.158}}\right) = 0.883$$

4.15

(a) P is lognormally distributed with

$\lambda_P = \lambda_C + \lambda_R + 2\lambda_V - \ln 2$

$= (\ln 1.8 - 0.5\text{x}0.2^2) + (\ln 2.3\text{x}10^{-3} - 0.5\text{x}0.1^2) + 2[\ln 120 - 0.5\text{x}\ln(1+0.45^2)] - \ln 2$

$= 0.568 + (-6.08) + 2(4.6953) - 0.693$

$= 3.186$

ξ_P =

$\sqrt{\xi_C^2 + \xi_R^2 + 4\xi_V^2} = \sqrt{0.2^2 + 0.1^2 + 4\times\ln(1+0.45^2)} = \sqrt{0.2^2 + 0.1^2 + 4\times 0.1844}$

$= 0.887$

(b) $P(P>30) = 1-\Phi(\frac{\ln 30 - 3.186}{0.887}) = 1-\Phi(0.243) = 0.596$

(c) C is lognormally distributed with

$\lambda_C = \ln 90 - 0.5\text{x}0.15^2 = 4.4886$

$\xi_C = 0.15$

P(Failure of antenna) = P(C<P) = P(C/P<1)

Define B = C/D

B also follows a lognormal distribution with

$\lambda_B = \lambda_C - \lambda_P = 4.4886 - 3.186 = 1.7$

$\xi_B = \sqrt{\xi_C^2 + \xi_P^2} = \sqrt{0.15^2 + 0.887^2} = 0.9$

Hence, P(Failure of antenna) = P(B<1)

$= \Phi(\frac{\ln 1 - \lambda_B}{\xi_B}) = \Phi(\frac{-1.7}{0.9}) = \Phi(-1.89) = 0.029$

(d) Mean rate of wind storm causing failure

= 1/5 x 0.029 = 0.0058

P(antenna failure in 25 years) = 1-P(no damaging storm in 25 years)

$= 1 - e^{-0.0058\text{x}25} = 0.135$

(e) P(at least two out of 5 antenna failures)

= 1 – P(X=0) – P(X=1)

$= 1 - 0.865^5 - 5(0.135)(0.865)^4$

= 0.138

4.16

(a) P(failure)=P(L>C)=P(C/L<1)=P(Z<1)

Z in LN with $\lambda_Z = \lambda_C - \lambda_L$

$$\zeta_Z = \sqrt{\zeta_C^2 + \zeta_L^2}$$

in which $\zeta_C = 0.2$

$$\lambda_C = \ln 20 - \frac{1}{2}(0.2)^2 = 2.259$$

$$\zeta_L = \sqrt{\ln(1+\delta^2)} = .294$$

$$\lambda_L = \ln 10 - \tfrac{1}{2}(0.294)^2 = 2.259$$

$$\therefore \lambda_Z = 2.976 - 2.259 = 0.717$$

$$\zeta_Z = \sqrt{0.2^2 + 0.294^2} = 0.356$$

$$\therefore P_F = \Phi(\frac{\ln 1 - 0.717}{0.356}) = \Phi(-2.014) = 1 - 0.978 = 0.022$$

(b) $T=C_1+C_2$

$E(T)=E(C_1)+E(C_2)=20+20=40$

$$Var(T) = Var(C_1) + Var(C_2) + 2\rho\sigma_{C_1}\sigma_{C_2} = 57.6$$

$$\delta_T = \frac{\sqrt{57.6}}{40} = 0.19$$

(c) some other distribution

4.17

(a) C=F+B

$$\mu_C = \mu_F + \mu_B = 20 + 30 = 50$$

$$\sigma_C = \sqrt{(0.2\times 20)^2 + (0.3\times 30)^2} = 9.85$$

(b) T=C_1+C_2=2 μ_C =0.197

$$\sigma_T = \sqrt{9.85^2 + 9.85^2 + 2\times 0.8\times 9.85^2} = 18.69$$

$$\therefore \delta_T = 18.69/100 = 0.187$$

(c) P(failure)=P(T<L)=P(T-L<0)

$$= \Phi(\frac{0-\mu_Z}{\sigma_Z})$$

$$= \Phi(\frac{-(100-50)}{\sqrt{18.69^2 + (.3\times 50)^2}}$$

$$= \Phi(\frac{-50}{23.95})$$

$$= 0.0184$$

4.18

(a) $F = 18 + \sum_{i=1}^{16} A_i$

$\mu_F = 18 + 16 \times 0.1 = 19.6$

$\sigma_F = \sqrt{(0.3 \times 0.1)^2 \times 16} = 0.12$

P(F>20)=1- $\Phi(\frac{20-19.6}{0.12}) = 0.00043$

(b)

i. P(no collapse)=P(C')=P(F<M)=P(F-M<0)=P(Z<0)

$\mu_Z = \mu_F - \mu_M = 19.6 - 20 = -0.4$

$\sigma_Z = \sqrt{(0.12)^2 + (0.01 \times 20)^2} = 0.233$

P(C')= $\Phi(\frac{0-(-0.4)}{0.233}) = 0.957$

ii. F=18+16A

$\mu_F = 18 + 16 \times 0.1 = 19.6$

$\sigma_F = \sqrt{(0.3 \times 0.1)^2 \times 16^2} = 0.48$

$\sigma_Z = \sqrt{(0.48)^2 + (0.01 \times 20)^2} = 0.52$

P(C')= $\Phi(\frac{0-(-0.4)}{0.52}) = 0.779$

4.19

(a) a is lognormal with mean 0.3g and c.o.v. of 25%
W = 200 kips
F = wa/g is also lognormal with
λ_F = ln 200 + λ_a = 5.298 – 1.204 = 4.094
$\xi_F = \xi_a$ = 0.25
R = frictional resistance = WC

Where C = coefficient of friction is lognormal with median 0.4 and a c.o.v. of 0.2

Hence R is lognormal with
λ_R = ln 200 + λ_C = ln200 + ln0.4 = 4.382
$\xi_R = \xi_C$ = 0.2

P(failure) = P(R<F)

$$= \Phi\left(\frac{-\lambda_R + \lambda_F}{\sqrt{\xi_R^2 + \xi_F^2}}\right) = \Phi\left(\frac{-4.382 + 4.094}{0.32}\right) = \Phi(-0.9) = 0.184$$

(b) P(none out of five tanks will fail)
= $(0.184)^5$ = 0.00021