2.1 $v = \sqrt{2} \times 120 \cos(\omega t + 30°) \Rightarrow V = 120\angle 30°$ V

$i = \sqrt{2} \times 10 \cos(\omega t - 30°) \Rightarrow I = 10 \angle -30°$ A

(a) $p(t) = |V||I|[\cos\varphi + \cos(2\omega t + \angle V + \angle I)]$

$= 600 + 1200 \cos 2\omega t$ W

$S = VI^* = 1200\angle 60° = P + jQ \Rightarrow P = 600\text{ W},\ Q = 1039$ VAr

(b) $Z = V/I = 12\angle 60° = 6 + j10.39 = R + jX \Rightarrow R = 6,\ X = 10.39$

2.2 (a) Using (2.3) we find $p_{max} = 1707 = |V||I|(\cos\varphi + 1)$ and $p_{min} = -293 = |V||I|(\cos\varphi - 1)$. Then, since $|V| = 100$, we get $|I| = 10$ and $\cos\varphi = \pm 45°$. Pick $\varphi = 45° \Rightarrow Z = 10\angle 45° = 7.07 + j7.07 = R + jX \Rightarrow R = 7.07,\ X = 7.07$

(b) $S = VI^* = Z|I|^2 = (7.07 + j7.07)\,10^2 \Rightarrow P = 707,\ Q = 707$

(c) For simplicity assume $i(t) = \sqrt{2}\,|I| \cos\omega t$. Then

$p_L(t) = v_L(t)\,i(t) = L\frac{di}{dt}\,i = -2\omega L|I|^2 \cos\omega t \sin\omega t = -\omega L|I|^2 \sin 2\omega t$

$P_{L\,max} = \omega L|I|^2 = 707 = Q$. Thus $P_{L\,max} = Q$. The same!

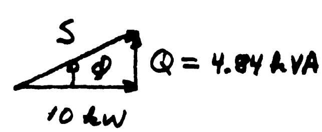
2.3 0.7 PF lagging $\Rightarrow \varphi = 45.57°$, $Q = 5.10$ MVAr

0.9 PF lagging $\Rightarrow \varphi = 25.84°$, $Q = 2.42$ MVAr. Capacitor must supply $5.10 - 2.42 = 2.68$ MVAr.

2.4 0.707 PF lagging $\Rightarrow S_{3\phi} = 200 + j200$ kVA. Cap supplies 50 kVAr. Resultant $S_{3\phi} = 200 + j150$ kVA $\Rightarrow$ PF $= 0.80$.

$|S| = \frac{|S_{3\phi}|}{3} = \frac{250 \times 10^3}{3} = |V||I| = \frac{440}{\sqrt{3}}|I| \Rightarrow |I| = 328$ A

2.5 0.9 PF lagging $\Rightarrow \varphi = 25.84°$



(a) $S = 10 + j\,4.84$ kVA

(b) $10 \times 10^3 = 416 \times |I| \times 0.9 \Rightarrow |I| = 26.71$ A

(c) Using (2.3), (or first principles) we get

$p(t) = 10 \times 10^3 + 11.11 \times 10^3 \cos(2\omega t + 25.84°)$

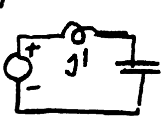
Note: the average value of $p(t)$ is 10 kW

2.6 Because system is balanced $V_{ab} = 208\angle 120°$, $V_{bc} = 208\angle 0°$. Using (2.17) or Fig 2.11, $V_{an} = 120\angle 90° \Rightarrow V_{bn} = 120\angle -30°$, $V_{cn} = 120\angle -150°$. Using per phase analysis, $I_a = 12\angle 105° \Rightarrow$ $I_b = 12\angle -15°$, $I_c = 12\angle -135°$.

2.7 $S = VI^* = V(YV)^* = Y^*|V|^2 = Y_C^* + Y_L^* + Y_R^*$
$= -j5 + j10 + 0.1 = 0.1 + j5$

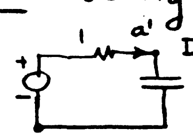
2.8 (a) Using loop or nodal analysis we find, after much work,
$I_a = 0.9123\angle -90.351°$, $I_b = 0.9123\angle -209.65°$, $I_c = 0.9929\angle 30°$.
(b) Using per phase analysis $I_a = 1\angle 0° / j1.1 = 0.9091\angle -90°$,
then, $I_b = 0.9091\angle -210°$, $I_c = 0.9091\angle 30°$.

2.9 Proceeding by analogy with 3ϕ, we note
$E_{ab} = E_{an} - E_{bn} = E_{an}(1 - e^{-j\pi/2}) = \sqrt{2}\, E_{an}\, e^{j\pi/4}$.
Thus $E_{an} = \frac{1}{\sqrt{2}} E_{ab} e^{-j\pi/4}$, and $E_{an}, E_{bn}, E_{cn}, E_{dn}$ form a pos. seq. set of 4ϕ voltages. Doing per phase (phase a) analysis we have

$\frac{1}{\sqrt{2}}\angle 45°$  $-j0.5 \Rightarrow I_a = \dfrac{\frac{1}{\sqrt{2}}\angle -45°}{j0.5} = \sqrt{2}\angle -135°$

Then $I_b = \sqrt{2}\angle -225°$, $I_c = \sqrt{2}\angle -315°$, $I_d = \sqrt{2}\angle -405°$

2.10 Using per phase circuit we find $I_a = 103.8\angle 41.5°$.

$\frac{240}{\sqrt{3}}$  $Z_\Delta = \frac{1}{3}\frac{1}{j\omega C} = -j0.884$

Then $|I_b| = 103.8$ A

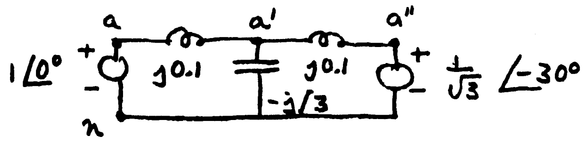
$V_{a'n} = Z_\Delta I_a = 91.76\angle -48.5° \Rightarrow |V_{a'b'}| = \sqrt{3}\cdot 91.76 = 158.9$ V

$S_{load} = V_{a'n} I_a^* = 9524\angle -90°$

$S_{load}^{3\phi} = 3\, S_{load} = 28574\angle -90°$ W

2.11 Assume pos. seq. operation. $V_{a''b''} = V_{a''n} - V_{b''n} = \sqrt{3}\, V_{a''n}\, e^{j\pi/6} \Rightarrow V_{a''n} = \frac{1}{\sqrt{3}} V_{a''b''}\, e^{-j\pi/6} = \frac{1}{\sqrt{3}} \angle -30^\circ$

Per Phase Ckt



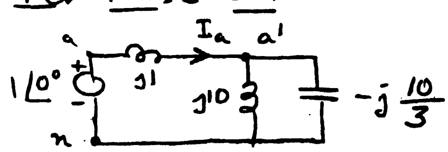
Using superposition & voltage divider law we get

$V_{a'n} = 0.899 \angle -10.89^\circ \Rightarrow V_{b'n} = 0.899 \angle -130.89^\circ$ and

$V_{c'n} = 0.899 \angle -250.89^\circ$. Then $V_{a'b'} = 1.557 \angle 19.11^\circ$

2.12 Assume pos. seq..

Per Phase Ckt



Combining parallel elements we have $Z_{\|} = -j5$. $I_a = 0.25 \angle 90^\circ$

$V_{a'n} = -j5 \cdot j0.25 = 1.25 \angle 0^\circ$

$V_{a'b'} = 2.165 \angle 30^\circ \Rightarrow I_{cap} = .2165 \angle 120^\circ$

$S_{load}^{3\phi} = 3\, V_{a'n}\, I_a^* = 0.3125 \angle -90^\circ$

2.13

(a) $V_{bc} = 208 \angle -120^\circ$, $V_{ca} = 208 \angle 120^\circ$

$V_{an} = \frac{208}{\sqrt{3}} \angle -30^\circ \Rightarrow I_a = 1.20 \angle -90^\circ$

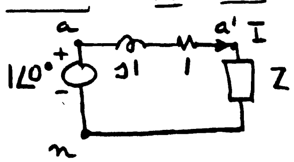
then $I_b = 1.20 \angle -210^\circ$, $I_c = 1.20 \angle -330^\circ$

(b) $V_{bc} = 208 \angle 120^\circ$, $V_{ca} = 208 \angle -120^\circ$

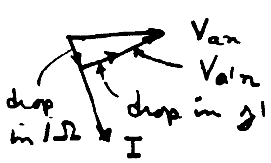
$V_{an} = \frac{208}{\sqrt{3}} \angle 30^\circ \Rightarrow I_a = 1.20 \angle -30^\circ$

$I_b = 1.20 \angle 90^\circ$, $I_c = 1.20 \angle -150^\circ$

2.14 Per Phase Ckt. Problem reduces to picking Z so that $|V_{a'n}| > |V_{an}|$. It helps to draw some phasor diagrams.

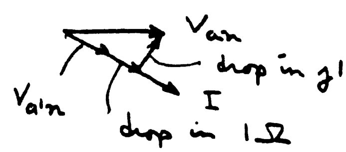


I. $Z = j\omega L$



Clearly $|V_{a'n}| < |V_{an}|$

II $Z = R$



Clearly $|V_{a'n}| < |V_{an}|$

III $Z = -j\frac{1}{\omega C}$

For example $Z = -j2$

$I = \frac{1}{\sqrt{2}} \angle 45^\circ$

$V_{a'n} = \sqrt{2} \angle -45^\circ$

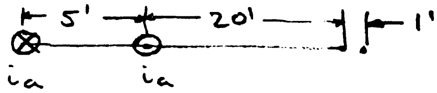
$|V_{a'n}| > |V_{an}|$

this is O.K.

2.15 Since $E_a+E_b+E_c=0$, neutrals are at same potential. $Z = E_a/I_a = \sqrt{2}\,\angle 55°$. For each Z, $S = VI^* = |V|^2/Z^*$. Thus

$$S^{3\phi} = \frac{(\sqrt{2})^2+1^2+1^2}{\sqrt{2}\,\angle -55°} = 2\sqrt{2}\,\angle 55°.$$

3.1



$\frac{\mu_0}{2\pi} = 2\times10^{-7}$, $|I_a| = 100$

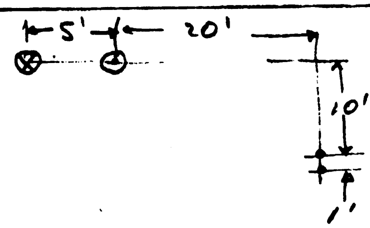
(Upward) flux linkages of telephone wires

$$\lambda = 2\times10^{-7}\left[\, i_a \ln\frac{26}{25} - i_a \ln\frac{21}{20}\right] = -.00957\times2\times10^{-7}\, i_a$$

$$v_{tel} = \frac{d\lambda}{dt} \Rightarrow V_{tel} = j\omega \times(-0.00957\times2\times10^{-7})\times 100 \quad V/\text{meter}$$

Since 1 mile = 1.609 km, $|V_{tel}| = 0.116$ V/mile

3.2

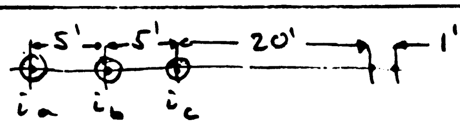


$$\lambda = 2\times10^{-7}\, i_a\left[\ln\frac{\sqrt{25^2+11^2}}{\sqrt{25^2+10^2}} - \ln\frac{\sqrt{20^2+11^2}}{\sqrt{20^2+10^2}}\right]$$

$$= -0.066294\times2\times10^{-7}\, i_a$$

$$\Rightarrow |V_{tel}| = 0.076 \text{ V/mile}$$

3.3



$$\lambda = 2\times10^{-7}\left[\, i_a \ln\frac{31}{30} + i_b \ln\frac{26}{25} + i_c \ln\frac{21}{20}\right]$$

$$= 2\times10^{-7}\left[0.0328\, i_a + 0.0392\, i_b + 0.0488\, i_c\right]$$

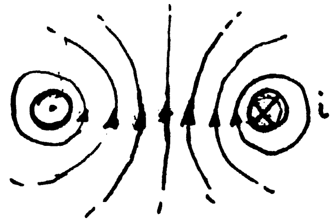
$$V_{tel} = j\omega\times2\times10^{-7}\left[0.0328\, I_a + 0.0392\, I_b + 0.0488\, I_c\right]$$

$$|V_{tel}| = 377\times2\times10^{-7}\times100\times\left|0.0328 + 0.0392\,\angle -120° + 0.0488\,\angle 120°\right|$$

$$= 1048\times10^{-7}\ V/\text{meter} = 0.1692\ V/\text{mile}$$

3.4 The telephone wires are transposed every 1000 ft. Cancellation occurs in all but 720' of line $\Rightarrow$ 0.0158 V.

3.5



The flux linkages due to the current in the left conductor is ≈

$$\lambda_1 = \frac{\mu_0}{2\pi} i \left[\frac{\mu_r}{4} + \ln \frac{D}{r}\right] = i \frac{\mu_0}{2\pi} \ln \frac{D}{r'}$$

Here we are neglecting partial flux linkages of the far (right) conductor. The current in the right conductor contributes an equal number of flux linkages. Thus (at least approximately),

$$l = 2\lambda_1 / i = \frac{\mu_0}{\pi} \ln \frac{D}{r'} = 4 \times 10^{-7} \ln \frac{D}{r'} \quad \text{H/meter}$$

3.6 If each conductor is hollow then there are no partial flux linkages and in (3.13) the term involving μ_r is absent. Then $r' = r$ and

$$l = \frac{\mu_0}{2\pi} \ln \frac{D}{r} \qquad \text{H/m}$$

3.7 The hint is misleading. A better hint for combining the 7 (unequal) inductances $l_1, l_2, \cdots, l_7$ would be to use the average inductance i.e. let $l_a = \frac{l_{av}}{7} = \frac{l_1 + l_2 + \cdots + l_7}{7^2}$ Returning to the problem, paralleling the steps which lead to (3.27) we have

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[\frac{i_a}{7}\left(\ln\frac{1}{d_{11}} + \ln\frac{1}{d_{12}} + \cdots + \ln\frac{1}{d_{17}}\right)\right.$$

$$+ \frac{i_b}{7}\left(\ln\frac{1}{d_{18}} + \ln\frac{1}{d_{19}} + \cdots + \ln\frac{1}{d_{1,21}}\right)$$

$$\left. + \frac{i_c}{7}\left(\ln\frac{1}{d_{1,15}} + \ln\frac{1}{d_{1,16}} + \cdots + \ln\frac{1}{d_{1,21}}\right)\right]$$

$$\approx \frac{\mu_0}{2\pi} i_a \ln \frac{D}{(d_{11} \cdots d_{17})^{1/7}} \Rightarrow l_1 = 7 \times \frac{\mu_0}{2\pi} \ln \frac{D}{(d_{11} \cdots d_{17})^{1/7}}$$