Chapter 1

1(c) Binding energy of a proton and neutron in $^{139}$La:

$$^{138}\text{La} + ^{1}\text{H} \rightarrow ^{139}\text{La} + \text{BE (neutron)}$$

$$^{138}\text{La} + ^{1}\text{H} \rightarrow ^{139}\text{La} + \text{BE (proton)}$$

$\text{BE (neutron)} = \left[ \frac{138}{57} \Delta + \Delta N - \frac{139}{57} \Delta \right] = \left[ -86524.681 + 8071.371 - (-87231.371) \right]$

$$= 778.0071 \text{ keV}$$

$\text{BE (proton)} = \left[ \frac{138}{56} \Delta + \Delta N - \frac{139}{57} \Delta \right] = \left[ -88261.631 + 7288.9705 - (-87231.371) \right]$

$$= 6257.7105 \text{ keV}$$

- The binding energy of the neutron is $8.779(0071) \text{ MeV}$ and the binding energy of the proton is $6.257(7105) \text{ keV}$.

(iii) The mean binding energy per nucleon is:

For $^{139}\text{La}$:

$$\text{BE per nucleon} = \frac{8.7780071}{57} \text{ keV/nucleon} = 154.393 \text{ keV/nucleon}$$

The mean binding energy is $8.378063 \text{ MeV/nucleon}$ (same number as Appendix B).

This value is slightly less than that given for the neutron but greater than that for the proton since the neutron is paired in La-139 whereas the proton is the odd one.

(iii) $^{139}\text{Pr}$ and $^{139}\text{Ce}$ have $\beta^+$ and EC transition energies of 2.11 and 0.28 MeV, respectively.

i.e., $^{139}\text{Pr} \rightarrow ^{139}\text{Ce} \rightarrow ^{139}\text{La}$

Now $E(\beta^+) = (\text{mass} \ 139\text{Pr} - \text{mass} \ 139\text{Ce}) \times 931.4944 - 2 \times 0.511 \text{ MeV}$ and

$E(\text{EC}) = (\text{mass} \ 139\text{Ce} - \text{mass} \ 139\text{La}) \times 931.4944 \text{ MeV}$ (using CODATA data).

$$E(\beta^+) = 2.11 \text{ MeV} \quad (1)$$

$$E(\text{EC}) = 0.28 \text{ MeV} \quad (2)$$

Mass excess of $^{139}\text{La}$ is $\frac{-87231.371}{57} \text{ keV}$ or in mass units:

$$\frac{-87231.371}{57} \text{ keV}$$
\[ 139 \Delta = \frac{-87.231371 \text{ meV}}{931.494 \text{ MeV / amu}} = -936.673 \text{ MeV} \]

Using Eq. (12): \[ \text{mass } ^{139} \text{Ce} = \frac{-28}{931.494} + 138.9065 \text{ u} = 138.9065 \text{ u} \]

and Eq. (1): \[ \text{mass } ^{59} \text{Pr} = \frac{2.111 + 2.0511}{931.494} + 138.9065 = 138.9101 \text{ u} \]

(CV) Consider the binding energy equation:

\[ B_{\text{bind}} = \sum Z \text{pm} + (A-2) \text{Mn} - M_{\text{A}2} \]

in terms of mass def:

\[ S_{\text{M}A_{2}} = \frac{B_{\text{bind}}}{931.494} = \sum Z \text{pm} + (A-2) \text{Mn} - M_{\text{A}2} \]

Given the definition of mass excess:

\[ M_{\text{A}2} = \Delta A + A \]

\[ S_{\text{M}A_{2}} = \left[ Z \text{pm} + (A-2) \text{Mn} - (\Delta A + A) \right] \]

\[ = \left[ 2Z \Delta H + 2A \Delta A + A - 2 \Delta n - Z - \Delta A - B \right] \]

\[ = \left[ 2Z \Delta H + (A-2) \Delta A - \Delta A \right] \]

(CV) For \(^{59}\text{Pr}\), \(A\) is odd \(\Rightarrow\) spin + parity obtained from the 59th proton

in the \(2d_{5/2}\) level \(\Rightarrow\) for \(d, \ell = 2\) : \(\ell^2\) parity and spin \(\frac{1}{2}\)

Similarly, for \(^{139}\text{Ce}\), the spin and parity are obtained from the

\(1f_{7/2}\) neutron in the \(1h_{11/2}\) level \(\Rightarrow\) for \(h, \ell = 5\) : \(-\) parity and spin \(\frac{3}{2}\)

(Actually \(\frac{3}{2}\))

For \(^{139}\text{La}\), the spin and parity are obtained from the 59th proton in the

\(1g_{9/2}\) level \(\Rightarrow\) for \(g, \ell = 4\) : \(+\) parity and spin \(\frac{3}{2}\)

However, the \(\Delta \ell = 3\) for the transition from the ground state for

\(^{139}\text{Pr}, \ell = \frac{1}{2}\) \(\rightarrow\) \(^{139}\text{Ce}, \ell = \frac{3}{2}\) : \(\Delta \ell = \frac{3}{2} - \frac{1}{2} = -3\) yes (There is a parity change)

\(^{59}\text{Pr}, \ell = \frac{1}{2}\) \(\rightarrow\) \(^{58}\text{Ce} , \ell = \frac{3}{2}\)

Hence the beta selection rules suggest a \(3^+\) for hidden type.

However, since we are near a magic number for \(^{139}\text{Ce} (83\text{rd} \text{neutron}) and\)
the fact that the spin prediction suggest a 3rd forbidden type, it is reasonable that a lower energy state may exist by promoting a neutron from a level of lower angular momentum into the higher angular momentum level so that a pair can be formed. Hence, for $^{139}$Ce, it appears that a neutron removed from a pair in the $2d_{\frac{3}{2}}$ level to form a pair with the $1h_{\frac{1}{2}}$ neutron; the odd particle is therefore a $2d_{\frac{3}{2}}$ neutron (implying a $+\frac{3}{2}$ parity and spin ground state). Alternatively, a neutron could have been promoted from the $3s_{\frac{1}{2}}$ state for $^{139}$Ce; however, this would have implied a greater angular momentum change for the $\beta$ decay transition ($\Delta J = \frac{5}{2} - \frac{1}{2} = 2$ no $\Rightarrow$ 2nd forbidden type), as well as for the EC transition.

Therefore, the most likely transitions are:

1) $^{139}$Pr

   & $\left(\frac{5}{2}^+\right)$ ground state

   & $\Delta J = \frac{5}{2} - \frac{3}{2} = 1$ (no $\Rightarrow$ 2nd forbidden type)

2) $^{139}$Ce

   & $\left(\frac{7}{2}^-\right)$ ground state

   & $\Delta J = \frac{3}{2} - \frac{5}{2} = -1$ (no $\Rightarrow$ allowed type)

Therefore, with reference to the above figure, for $^{139}$Ce, the $57^{th}$ neutron is in the $1g_{\frac{7}{2}}$ level (ground state $\frac{7}{2}^-$) and in the energy level diagram the next state is $2d_{\frac{3}{2}}$ (i.e., $1^{st}$ excited state $\frac{7}{2}^-$). Hence, the $\Delta J$ change for the EC transition is less between the ground state of $^{139}$Ce and the $1^{st}$ excited state of $^{139}$La, than between the ground state and the $1^{st}$ excited state of the $^{139}$La, then the $\Delta J$ change for this EC transition is excited state. Note that the $\Delta J$ change for this EC transition is $\frac{3}{2} - \frac{7}{2} = -1$ no, which according to $\beta$ decay systematics is an allowed type transition. Hence, this transition is likely by EC decay since the theory of EC decay resembles that of $\beta$ decay (i.e., it only involves the $\psi$ wave function of the atomic electron). Further, $^{139}$Ce undergoes an EC decay rather than $\beta$ decay because the mass which equals 0.28 MeV is less than the threshold 1.022 MeV for $\beta$ decay.
Using the binding energy per nucleon (Appendix B) for $^{139}$Pr, $^{139}$Ce, $^{139}$La, the corresponding binding energies $B$ are:

\[
\begin{align*}
B_{\text{Pr}} &= 8349.482 \text{ (kev/nucleon)} \times 139 \text{ nucleons} = 1160.578 \text{ MeV} \\
B_{\text{Ce}} &= 8370.428 \text{ (kev/nucleon)} \times 139 \text{ nucleons} = 1163.489 \text{ MeV} \\
B_{\text{La}} &= 8358.063 \text{ (kev/nucleon)} \times 139 \text{ nucleons} = 1164.551 \text{ MeV}
\end{align*}
\]

Given the Weissacker formula for odd A nuclei:

\[
B(\Lambda_{1/2}) = 14.1\Lambda - 13.1\Lambda^{2/3} - 0.582(2\Lambda-1)\Lambda^{1/3} - 18(\Lambda-2)^2\Lambda^{-1}
\]

Hence, the binding energies are:

\[
\begin{align*}
B_{\text{Pr}} &= 139\ \text{Pr} \\
B_{\text{Ce}} &= 139\ \text{Ce} \\
B_{\text{La}} &= 139\ \text{La}
\end{align*}
\]

The discrepancy in binding energies is due to the fact that the constants in the Weissacker formula are derived over a large range of $A$ values. Thus, we extrapolate the binding energies by fitting the constants to the specific mass data where:

\[
B = aZ^2 + bZ + c
\]

for the matrix problem

\[
A \vec{x} = \vec{b}
\]

where

\[
A = \begin{bmatrix}
59^2 & 59 & 1 \\
58^2 & 58 & 1 \\
57^2 & 57 & 1
\end{bmatrix}, \quad \vec{x} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} -1160.578 \\ -1163.489 \\ -1164.551 \end{bmatrix}
\]

can be solved by Maple (see attached) yielding the solution

\[
A = 0.92, \quad b = -105.2459449, \quad c = 1831.310530
\]

See attached plot of $-B$ (MeV) versus $Z$.
with(linalg):
b := vector([-1160.578, -1163.489, -1164.551]);
b := [-1160.578, -1163.489, -1164.551]
A := matrix(3, 3, [59^2, 59, 1, 58^2, 58, 1, 57^2, 57, 1]);
A := [3481  59  1]
  [3364  58  1]
  [3249  57  1]
x := linsolve(A, b);
x := [0.9244995555, -105.2554489, 1831.310530]
a := 0.9244995555
b := -105.2554489
c := 1831.310530
plot((a*Z^2 + b*Z + c), Z = 53..59);# (KE (MeV) vs Z)
2. The threshold energy $T_m$ for a particle of mass $m$ striking a nucleus of mass $M$ with a reaction energy $Q$ is:

$$T_m = -Q \frac{(M+m)}{M}$$

(i) $^{204}Pb + \alpha \rightarrow ^{203}Pb + ^{1}n$

$$Q = (\Delta ^{204}Pb - \Delta ^{203}Pb - \Delta n)$$

where $\Delta$ is the mass excess in keV.

$$Q = (-25109.735 - (-24786.57) - 8071.3171) = -8394.482 \text{ keV}$$

This reaction is endothermic and the threshold energy necessary to make this reaction go is $Q$ (since $m=0$ for a $\alpha$-ray).

$$T_m = -Q = 8.394 \text{ meV}$$

(ii) $^{109}Ag + ^4He \rightarrow ^{1}p + ^{112}Cd$

$$Q = (\Delta ^{109}Ag + \Delta ^{4}He - \Delta ^{112}Cd - \Delta ^{1}He) \text{ keV}$$

where 2 electron masses were added to each side of the equation.

$$Q = ( - 88722.669 + 2424.91565 - (-90580.518) - 2488.7705) \text{ keV} = -3006.206 \text{ keV}$$

This reaction is also endothermic. Therefore, the threshold energy is:

$$T_m = -Q \frac{(M+m)}{M} = 3.006206 \left( \frac{108.904752 + 4.001506}{108.904752} \right) = 3.117 \text{ meV}$$

Here $m = 4.00260325 u - 2 \times \frac{511066 \text{ MeV}}{931.494 \text{ MeV/u}} = 4.001506 u$

The atomic masses are taken from Appendix B and $m$ is the mass of the He nucleus ($\alpha$ particle).
\[
\begin{align*}
\text{iii)} & \quad ^{135}\text{La} + ^{0}\text{n} \rightarrow ^{139}\text{La} + ^{8}\text{n} \\
Q &= \left[ \Delta^{135}\text{La} + \Delta^{0}\text{n} - \Delta^{139}\text{La} \right] \text{keV} \\
&= -86524.681 + 8071.317 - (-97231.371) \text{ keV} = 8778.0071 \text{ keV} \\
&\approx 8.778 \text{ MeV}
\end{align*}
\]

Hence this reaction is exoergic (\(Q\) positive) and therefore the reaction does not require a threshold energy for this reaction to occur (same for all \((n,γ)\) and some \((n,\text{fission})\))