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CHAPTER 1

LINEAR EQUATIONS AND MATHEMATICAL CONCEPTS

EXERCISES 1.1

1. $3x + 1 = 4x - 5$

$$1 = x - 5$$

$$x = 6$$

conditional equation

3. $5(x + 1) + 2(x - 1) = 7x + 6$

$$5x + 5 + 2x - 2 = 7x + 6$$

$$7x + 3 = 7x + 6$$

contradiction

5. $4(x + 3) = 2(2x + 5)$

$$4x + 12 = 4x + 10$$

contradiction

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7. $5x - 3 = 17$

$$5x = 20$$

$$x = 4$$

9. $2x = 4x - 10$

$$2x - 4x = -10$$

$$-2x = -10$$

$$x = 5$$

11. $4x - 5 = 6x - 7$

$$-5 + 7 = 6x - 4x$$

$$2 = 2x$$

$$1 = x$$

13. $0.6x = 30$

$$x = 30/0.60 = 50$$

15. $2/3 = (4/5)x - (1/3)$ multiply by 15 to eliminate fractions

$$1(2/3) = 15 \{(4/5)x - (1/3)\}$$

$$10 = 12x - 5$$

$$15 = 12x$$

$$5/4 = x$$

17. $5(x - 4) = 2x + 3(x - 7)$

$$5x - 20 = 2x + 3x - 21$$

$$5x - 20 = 5x - 21$$

No solution

19. $3s - 4 = 2s + 6$

$$s - 4 = 6$$

$$s = 10$$

21. $7t + 2 = 4t + 11$

$$7t - 4t = 11 - 2$$

$$3t = 9$$

$$t = 3$$

23. $4(x + 1) + 2(x - 3) = 7(x - 1)$

$$4x + 4 + 2x - 6 = 7x - 7$$

$$6x - 2 = 7x - 7$$

$$6x - 7x = -7 + 2$$

$$x = -5$$

$$x = 5$$

25. $\frac{x + 8}{2x - 5} = 2$ multiply by $2x - 5$ to eliminate the fraction

$$(x + 8) = 2(2x - 5)$$

$$x + 8 = 4x - 10$$

$$8 + 10 = 4x - x$$

$$18 = 3x$$

$$6 = x$$

(Check the result. Multiplication by a factor such as $2x - 5$ can introduce an extraneous solution.)

27. $8 - \{4[x - (3x - 4) - x] + 4\} = 38 - \{4[x - (3x - 4) - x] + 4\}$
 $= 3(x + 2)$

$$8 - \{4[x - 3x + 4 - x] + 4\} = 3x + 6$$

$$8 - \{4[-3x + 4] + 4\} = 3x + 6$$

$$8 - \{-12x + 16 + 4\} = 3x + 6$$

$$8 - \{-12x + 20\} = 3x + 6$$

$$8 + 12x - 20 = 3x + 6$$

$$12x - 12 = 3x + 6$$

$$9x = 18$$

$$x = 2$$

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29. $6x - 3y = 9$ for x

$$6x = 3y + 9$$

$$x = \frac{3y + 9}{6} = \frac{1}{2}y + \frac{3}{2}$$

31. $3x + 5y = 15$

$$5y = 15 - 3x$$

$$y = \frac{(15 - 3x)}{5}$$

$$y = 3 - \left(\frac{3}{5}\right)x$$

33. $V = LWH$

$$\frac{V}{LH} = W$$

35. $Z = \frac{(x - \mu)}{\sigma}$

$$Z\sigma = x - \mu$$

$$x = Z\sigma + \mu$$

37. Let x = monthly installment (\$).

Since Sally paid \$300 down, she owes $\$1300 - \$300 = \$1000$.

Therefore, $5x = 1000$ or $x = \$200$ is the monthly installment.

39. The consumption function is $C(x) = mx + b$. The slope is the "marginal propensity to consume." Therefore, $C(x) = 0.75x + b$.

The disposable income, $x = 2$, when consumption is $y = 11$ yields

$11 = (0.75)(2) + b$ and $b = 9.5$. The consumption function is

$C(x) = 0.75x + 9.5$.

41. a) $d = 4.5(2) = 9$ miles

b) $18 = 4.5t$ and $t = 18/4.5 = 4$ seconds

43. The tax is 6.2% or 0.062 in decimal form, so $T = 0.062x$, where x is $0 \leq x \leq 87,000$.

45. a) $BSA = 1321 + (0.3433)(20,000) = 8187 \text{ cm}^2$

b) $10,325 = 1321 + (0.3433)(Wt)$

$$9004 = (0.3433)(Wt)$$

$$9004/0.3433 = 26,228 \text{ g} = 26.2 \text{ kg}$$

EXERCISES 1.2

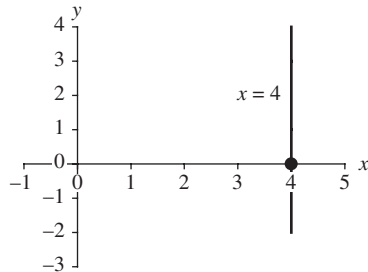
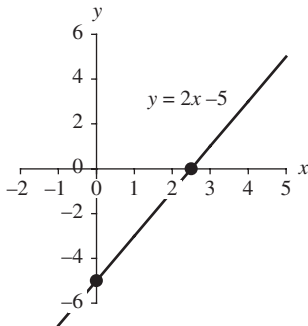
1. Setting $y = 0$ determines the x -intercept and setting $x = 0$ determines the y -intercept.

- a) $5x - 3y = 15$ x -intercept 3, y -intercept -5
- b) $y = 4x - 5$ x -intercept $5/4$, y -intercept -5
- c) $2x + 3y = 24$ x -intercept 12, y -intercept 8
- d) $9x - y = 18$ x -intercept 2, y -intercept -18
- e) $x = 4$ x -intercept 4, no y -intercept (vertical line)
- f) $y = -2$ no x -intercept (horizontal line), y -intercept -2

3. The slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$

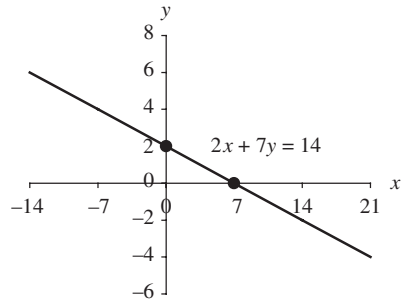
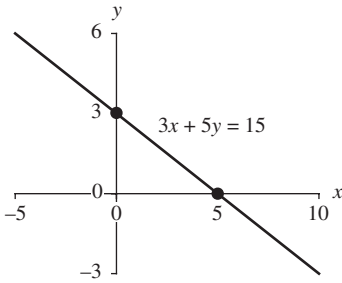
- a) $(3, 6)$ and $(-1, 4)$ $m = \frac{4 - 6}{-1 - 3} = \frac{-2}{-4} = \frac{1}{2}$
- b) $(1, 6)$ and $(2, 11)$ $m = \frac{11 - 6}{2 - 1} = \frac{5}{1} = 5$
- c) $(6, 3)$ and $(12, 7)$ $m = \frac{7 - 3}{12 - 6} = \frac{4}{6} = \frac{2}{3}$
- d) $(2, 3)$ and $(2, 7)$ $m = \frac{7 - 3}{2 - 2} = \frac{4}{0}$ undefined
- e) $(2, 6)$ and $(5, 6)$ $m = \frac{6 - 6}{5 - 2} = \frac{0}{3} = 0$
- f) $(5/3, 2/3)$ and $(10/3, 1)$ $m = \frac{1 - 2/3}{10/3 - 5/3} = \frac{1/3}{5/3} = \frac{1}{5}$

5. a) x -intercept $5/2$ and y -intercept -5 b) x -intercept 4 and no y -intercept



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- c) x -intercept 5 and y -intercept 3 d) x -intercept 7 and y -intercept 2



7. a) $y = (5/3)x + 2$ and $5x - 3y = 10$; the slope of the first line is $5/3$. Solving for y in the second equation yields $y = (5/3)x - (10/3)$. This slope is also $5/3$. The slopes are both $(5/3)$ so the lines are parallel (with different intercepts).
- b) $6x + 2y = 4$ and $y = (1/3)x + 1$. The slope of the second line is easily determined (line in slope intercept form) as $1/3$. Again, solve for y in the first equation to determine $y = -3x + 2$. The slope is -3 . The slopes are negative reciprocals; the lines are perpendicular.
- c) $2x - 3y = 6$ and $4x - 6y = 15$. Solving for y in each equation, one determines that $y = (2/3)x - 2$ and $y = (2/3)x - (5/2)$. These lines have the same slope (and different intercepts) making them parallel.
- d) $y = 5x - 4$ and $3x - y = 4$. The slope of the first line is 5 and solving for y in the second equation, ($y = 3x - 4$), the slope is 3. These slopes are neither the same nor negative reciprocals. They are neither parallel nor perpendicular.
- e) $y = 5$ is a horizontal line while $x = 3$ is a vertical line. The two lines are perpendicular.
9. A linear equation has a single x -intercept except for $y = 0$ (the x -axis) with an infinite number of x -intercepts. Any horizontal line except $y = 0$ has no x -intercepts. Generally, lines do not have more than one y -intercept. The exception is $x = 0$ (the y -axis) with an infinite number of y -intercepts. Any vertical line with the exception of $x = 0$ has no y -intercepts.

11. The ordered pairs of “time” and “machine value” are (0, 75,000) and (9, 21,000), respectively. The slope is

$m = \frac{21,000 - 75,000}{9 - 0} = \frac{-54,000}{9} = -6000$. The y-intercept is the purchase price, \$75,000. The equation to model the straight-line depreciation is $V(t) = -6000t + 75,000$, where $V(t)$ is the machine value (\$) at time t .

13. The ordered pairs (gallons, miles) are (7, 245) and (12, 420).

The slope is $\frac{420 - 245}{12 - 7} = \frac{175}{5} = 35$ with x gallons and y miles.

Use either pair with the point slope-formula.

Therefore, $y - 245 = 35(x - 7)$ or $y = 35x$.

15. Total cost reflects both fixed and variable costs. The fixed cost is monthly rent (\$1100). The variable cost is $5x$, where x is monthly production. Therefore, total cost is $C(x) = 1100 + 5x$.

17. a) Here, the fixed cost is \$50/day and variable cost \$0.30/mile. To rent the car for a single day costs \$50 to which the mileage cost must be added. The cost is $C(x) = 50 + 0.30x$.
- b) If a person has \$110 for rental, the equation to solve for the travel distance is $110 = 50 + 0.30x$. Solving yields,

$$60 = 0.30x$$

$$\frac{60}{0.30} = x$$

$$200 = x$$

The person can rent the car and travel 200 miles with \$110.

19. Since R is to be a function of C , the ordered pairs are (C, R) . The two ordered pairs are (70, 84) and (40, 48). The slope is $\frac{48 - 84}{40 - 70} = \frac{36}{30} = \frac{6}{5}$. Using either pair with the slope to yield $R - 84 = (6/5)(C - 70)$ or $R = (6/5)C$.