

## Chapter 2

# Signal and Linear System Analysis

### 2.1 Problem Solutions

#### Problem 2.1

- a. For the single-sided spectra, write the signal as

$$\begin{aligned}x_1(t) &= 10 \cos(4\pi t + \pi/8) + 6 \sin(8\pi t + 3\pi/4) \\&= 10 \cos(4\pi t + \pi/8) + 6 \cos(8\pi t + 3\pi/4 - \pi/2) \\&= 10 \cos(4\pi t + \pi/8) + 6 \cos(8\pi t + \pi/4) \\&= \operatorname{Re} \left[ 10e^{j(4\pi t + \pi/8)} + 6e^{j(8\pi t + \pi/4)} \right]\end{aligned}$$

For the double-sided spectra, write the signal in terms of complex exponentials using Euler's theorem:

$$\begin{aligned}x_1(t) &= 5 \exp[j(4\pi t + \pi/8)] + 5 \exp[-j(4\pi t + \pi/8)] \\&\quad + 3 \exp[j(8\pi t + 3\pi/4)] + 3 \exp[-j(8\pi t + 3\pi/4)]\end{aligned}$$

The spectra are plotted in Fig. 2.1.

- b. Write the given signal as

$$x_2(t) = \operatorname{Re} \left[ 8e^{j(2\pi t + \pi/3)} + 4e^{j(6\pi t + \pi/4)} \right]$$

to plot the single-sided spectra. For the double-side spectra, write it as

$$x_2(t) = 4e^{j(2\pi t + \pi/3)} + 4e^{-j(2\pi t + \pi/3)} + 2e^{j(6\pi t + \pi/4)} + 2e^{-j(6\pi t + \pi/4)}$$

The spectra are plotted in Fig. 2.2.

c. Change the sines to cosines by subtracting  $\pi/2$  from their arguments to get

$$\begin{aligned}x_3(t) &= 2 \cos(4\pi t + \pi/8 - \pi/2) + 12 \cos(10\pi t - \pi/2) \\&= 2 \cos(4\pi t - 3\pi/8) + 12 \cos(10\pi t - \pi/2) \\&= \operatorname{Re} \left[ 2e^{j(4\pi t - 3\pi/8)} + 12e^{j(10\pi t - \pi/2)} \right] \\&= e^{j(4\pi t - 3\pi/8)} + e^{-j(4\pi t - 3\pi/8)} + 6e^{j(10\pi t - \pi/2)} + 6e^{-j(10\pi t - \pi/2)}\end{aligned}$$

Spectral plots are given in Fig. 2.3.

d. Use a trig identity to write

$$3 \sin(18\pi t + \pi/2) = 3 \cos(18\pi t)$$

and get

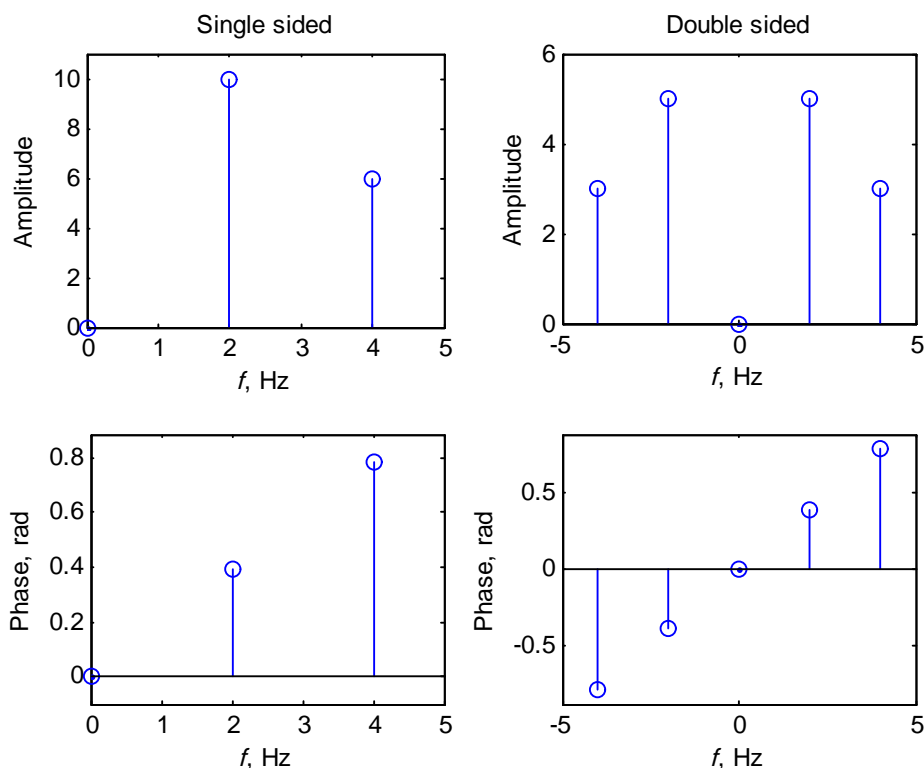
$$\begin{aligned}x_4(t) &= 2 \cos(7\pi t + \pi/4) + 3 \cos(18\pi t) \\&= \operatorname{Re} \left[ 2e^{j(7\pi t + \pi/4)} + 3e^{j18\pi t} \right] \\&= e^{j(7\pi t + \pi/4)} + e^{-j(7\pi t + \pi/4)} + 1.5e^{j18\pi t} + 1.5e^{-j18\pi t}\end{aligned}$$

From this it is seen that the single-sided amplitude spectrum consists of lines of amplitudes 2 and 3 at frequencies of 3.5 and 9 Hz, respectively, and the phase spectrum consists of a line of height  $\pi/4$  at 3.5 Hz. The double-sided amplitude spectrum consists of lines of amplitudes 1, 1, 1.5, and 1.5 at frequencies of 3.5, -3.5, 9, and -9 Hz, respectively. The double-sided phase spectrum consists of lines of heights  $\pi/4$  and  $-\pi/4$  at frequencies 3.5 Hz and -3.5 Hz, respectively.

e. Use  $\sin(2\pi t) = \cos(2\pi t - \pi/2)$  to write

$$\begin{aligned}x_5(t) &= 5 \cos(2\pi t - \pi/2) + 4 \cos(5\pi t + \pi/4) \\&= \operatorname{Re} \left[ 5e^{j(2\pi t - \pi/2)} + 4e^{j(5\pi t + \pi/4)} \right] \\&= 2.5e^{j(2\pi t - \pi/2)} + 2.5e^{-j(2\pi t - \pi/2)} + 2e^{j(5\pi t + \pi/4)} + 2e^{-j(5\pi t + \pi/4)}\end{aligned}$$

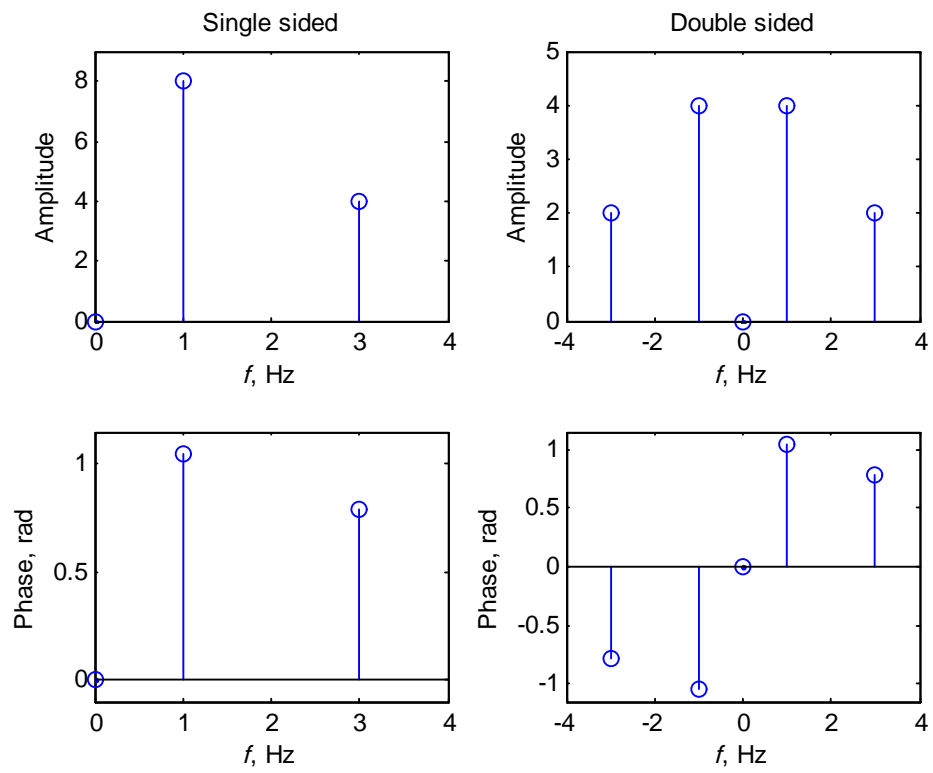
From this it is seen that the single-sided amplitude spectrum consists of lines of amplitudes 5 and 4 at frequencies of 1 and 2.5 Hz, respectively, and the phase spectrum consists of lines of heights  $-\pi/2$  and  $\pi/4$  at 1 and 2.5 Hz, respectively. The double-sided amplitude spectrum consists of lines of amplitudes 2.5, 2.5, 2, and 2 at frequencies of 1, -1, 2.5, and -2.5 Hz, respectively. The double-sided phase spectrum consists of lines of heights  $-\pi/2$ ,  $\pi/2$ ,  $\pi/4$ , and  $-\pi/4$  at frequencies of 1, -1, 2.5, and -2.5 Hz, respectively.

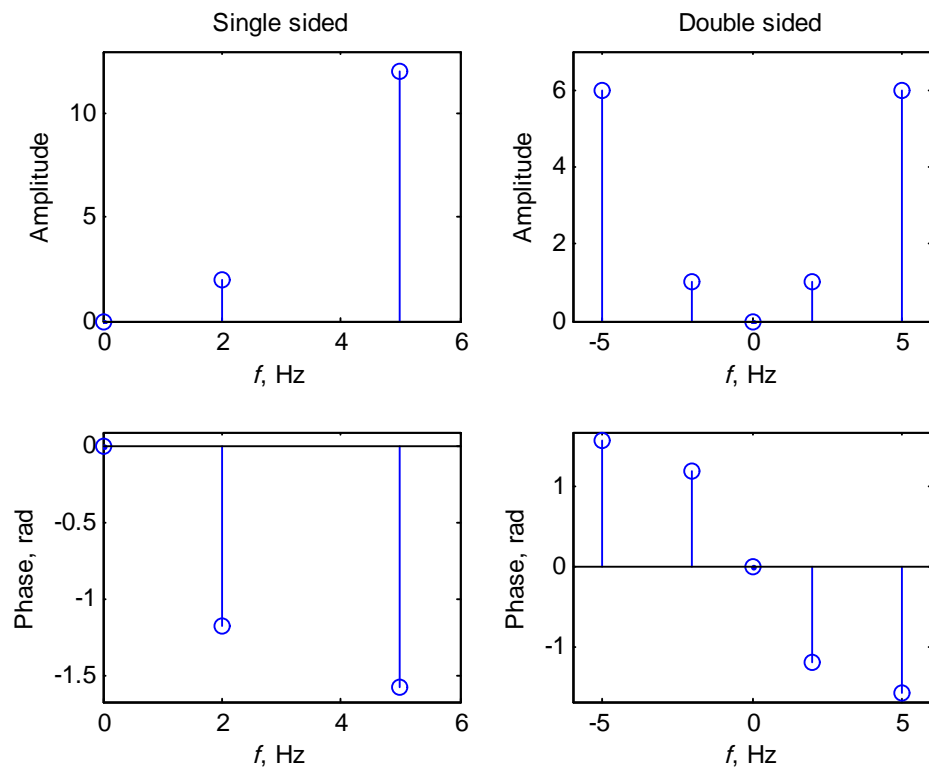


f. Use  $\sin(10\pi t + \pi/6) = \cos(10\pi t + \pi/6 - \pi/2) = \cos(10\pi t - \pi/3)$  to write

$$\begin{aligned}
 x_6(t) &= 3 \cos(4\pi t + \pi/8) + 4 \cos(10\pi t - \pi/3) \\
 &= \operatorname{Re} \left[ 3e^{j(4\pi t + \pi/8)} + 4e^{j(10\pi t - \pi/3)} \right] \\
 &= 1.5e^{j(4\pi t + \pi/8)} + 1.5e^{-j(4\pi t + \pi/8)} + 2e^{j10\pi t - \pi/3} + 2e^{-j(10\pi t - \pi/3)}
 \end{aligned}$$

From this it is seen that the single-sided amplitude spectrum consists of lines of amplitudes 3 and 4 at frequencies of 2 and 5 Hz, respectively, and the phase spectrum consists of lines of heights  $\pi/8$  and  $-\pi/3$  at 2 and 5 Hz, respectively. The double-sided amplitude spectrum consists of lines of amplitudes 1.5, 1.5, 2, and 2 at frequencies of 2, -2, 5, and -5 Hz, respectively. The double-sided phase spectrum consists of lines of heights  $\pi/8$ ,  $-\pi/8$ ,  $-\pi/3$ , and  $\pi/3$  at frequencies of 2, -2, 5, and -5 Hz, respectively.





**Problem 2.2**

By noting the amplitudes and phases of the various frequency components from the plots, the result is

$$\begin{aligned}x(t) &= 4e^{j(8\pi t + \pi/2)} + 4e^{-j(8\pi t + \pi/2)} + 2e^{j(4\pi t - \pi/4)} + 2e^{-j(4\pi t - \pi/4)} \\&= 8 \cos(8\pi t + \pi/2) + 4 \cos(4\pi t - \pi/4) \\&= -8 \sin(8\pi t) + 4 \cos(4\pi t - \pi/4)\end{aligned}$$

**Problem 2.3**

- a. Not periodic because  $f_1 = 1/\pi$  Hz and  $f_2 = 3$  Hz are not commensurable.
- b. Periodic. To find the period, note that

$$\frac{6\pi}{2\pi} = 3 = n_1 f_0 \quad \text{and} \quad \frac{30\pi}{2\pi} = 15 = n_2 f_0$$

Therefore

$$\frac{15}{3} = \frac{n_2}{n_1}$$

Hence, take  $n_1 = 1$ ,  $n_2 = 5$ , and  $f_0 = 3$  Hz (we want the largest possible value for  $f_0$  with  $n_1$  and  $n_2$  integer-valued).

- c. Periodic. Using a similar procedure as used in (b), we find that  $n_1 = 4$ ,  $n_2 = 21$ , and  $f_0 = 0.5$  Hz.
- d. Periodic. Using a similar procedure as used in (b), we find that  $n_1 = 4$ ,  $n_2 = 7$ ,  $n_3 = 11$ , and  $f_0 = 0.5$  Hz.
- e. Periodic. We find that  $n_1 = 17$ ,  $n_2 = 18$ , and  $f_0 = 0.5$  Hz.
- f. Periodic. We find that  $n_1 = 2$ ,  $n_2 = 3$ , and  $f_0 = 0.5$  Hz.
- g. Periodic. We find that  $n_1 = 7$ ,  $n_2 = 11$ , and  $f_0 = 0.5$  Hz.
- h. Not periodic. The frequencies of the separate terms are incommensurable.
- i. Periodic. We find that  $n_1 = 19$ ,  $n_2 = 21$ , and  $f_0 = 0.5$  Hz.
- j. Periodic. We find that  $n_1 = 6$ ,  $n_2 = 7$ , and  $f_0 = 0.5$  Hz.

**Problem 2.4**

- a. The single-sided amplitude spectrum consists of a single line of amplitude 5 at 6 Hz and the phase spectrum consists of a single line of height  $-\pi/6$  rad at 6 Hz. The double-sided amplitude spectrum consists of lines of amplitude 2.5 at frequencies  $\pm 6$  Hz. The double-sided phase spectrum consists of a line of height  $\pi/6$  at -6 Hz and a line of height  $-\pi/6$  at 6 Hz.
- b. Write the signal as

$$x_2(t) = 3 \cos(12\pi t - \pi/2) + 4 \cos(16\pi t)$$

From this it is seen that the single-sided amplitude spectrum consists of lines of heights 3 and 4 at frequencies 6 and 8 Hz, respectively, and the single-sided phase spectrum consists of a line of height  $-\pi/2$  radians at frequency 6 Hz (the phase at 8 Hz is 0). The double-sided amplitude spectrum consists of lines of height 1.5 and 2 at frequencies of 6 and 8 Hz, respectively, and lines of height 1.5 and 2 at frequencies  $-6$  and  $-8$  Hz, respectively. The double-sided phase spectrum consists of a line of height  $-\pi/2$  radians at frequency 6 Hz and a line of height  $\pi/2$  radians at frequency  $-6$  Hz.

- c. Use the trig identity  $\cos x \cos y = 0.5 \cos(x + y) + 0.5 \cos(x - y)$  to write

$$x_3(t) = 2 \cos 20\pi t + 2 \cos 4\pi t$$

From this we see that the single-sided amplitude spectrum consists of lines of height 2 at 2 and 10 Hz, and the single-sided phase spectrum is 0 at these frequencies. The double-sided amplitude spectrum consists of lines of height 1 at frequencies of  $-10$ ,  $-2$ , 2, and 10 Hz. The double-sided phase spectrum is 0.

- d. Use trig identities to get

$$\begin{aligned} x_4(t) &= 4 \sin(2\pi t) [1 + \cos(10\pi t)] \\ &= 4 \sin(2\pi t) - 2 \sin(8\pi t + \pi) + 2 \sin(12\pi t) \\ &= 4 \cos(2\pi t - \pi/2) + 2 \cos(8\pi t + \pi/2) + 2 \cos(12\pi t - \pi/2) \\ &= \operatorname{Re} \left[ 4e^{j(2\pi t - \pi/2)} + 2e^{j(8\pi t + \pi/2)} + 2e^{j(12\pi t - \pi/2)} \right] \\ &= 2e^{j(2\pi t - \pi/2)} + 2e^{-j(2\pi t - \pi/2)} + e^{j(8\pi t + \pi/2)} + e^{-j(8\pi t + \pi/2)} + e^{j(12\pi t - \pi/2)} + e^{-j(12\pi t - \pi/2)} \end{aligned}$$

From this we see that the single-sided amplitude spectrum consists of lines of heights 4, 2, and 2 at frequencies 1, 4, and 6 Hz, respectively and the single-sided phase spectrum is  $-\pi/2$  radians at 1 and 6 Hz and  $\pi/2$  radians at 4 Hz. The double-sided amplitude spectrum

consists of lines of height 2 at frequencies of 1 and  $-1$  Hz and of height 1 at frequencies of 4,  $-4$ , 6, and  $-6$  Hz. The double-sided phase spectrum is  $\pi/2$  radians at  $-1$ , 4, and  $-6$  Hz and  $-\pi/2$  radians at 1,  $-4$ , and 6 Hz.

- e. Clearly, from the form of the cosine sum, the single-sided amplitude spectrum has lines of heights 1 and 7 at frequencies of 3 and 15 Hz, respectively. The single-sided phase spectrum is zero. The double-sided amplitude spectrum has lines of heights 0.5, 0.5, 3.5, and 3.5 at frequencies of 3,  $-3$ , 15, and  $-15$  Hz, respectively. The double-sided phase spectrum is zero.
- f. The single-sided amplitude spectrum has lines of heights 1 and 9 at frequencies of 2 and 10.5 Hz, respectively. The single-sided phase spectrum is  $-\pi/2$  radians at 10.5 Hz and 0 otherwise. The double-sided amplitude spectrum has lines of heights 0.5, 0.5, 4.5, and 4.5 at frequencies of 2,  $-2$ , 10.5, and  $-10.5$  Hz, respectively. The double-sided phase spectrum is  $\pi/2$  radians at  $-10.5$  Hz and  $-\pi/2$  radians at 10.5 Hz and 0 otherwise.
- g. Convert the sine to a cosine by subtracting  $\pi/2$  from its argument. It then follows that the single-sided amplitude spectrum is 2, 1, and 6 at frequencies of 2, 3, and 8.5 Hz and 0 otherwise. The single-sided phase spectrum is  $-\pi/2$  radians at 8.5 Hz and 0 otherwise. The double-sided amplitude spectrum is 1, 1, 0.5, 0.5, 3, and 3 at frequencies of  $-2$ , 2,  $-3$ , 3,  $-8.5$ , and 8.5 Hz, respectively, and 0 otherwise. The double-sided phase spectrum is  $\pi/2$  radians at a frequency of  $-8.5$  Hz and  $-\pi/2$  radians at a frequency of 8.5 Hz. It is 0 otherwise.

**Problem 2.5**

- a. This function has area

$$\begin{aligned} \text{Area} &= \int_{-\infty}^{\infty} \epsilon^{-1} \left[ \frac{\sin(\pi t/\epsilon)}{(\pi t/\epsilon)} \right]^2 dt \\ &= \int_{-\infty}^{\infty} \left[ \frac{\sin(\pi u)}{(\pi u)} \right]^2 du = 1 \end{aligned}$$

where a tabulated integral has been used for  $\text{sinc}^2 u$ . A sketch shows that no matter how small  $\epsilon$  is, the area is still 1. With  $\epsilon \rightarrow 0$ , the central lobe of the function becomes narrower and higher. Thus, in the limit, it approximates a delta function.



b. The area for the function is

$$\text{Area} = \int_{-\infty}^{\infty} \frac{1}{\epsilon} \exp(-t/\epsilon) u(t) dt = \int_0^{\infty} \exp(-u) du = 1$$

A sketch shows that no matter how small  $\epsilon$  is, the area is still 1. With  $\epsilon \rightarrow 0$ , the function becomes narrower and higher. Thus, in the limit, it approximates a delta function.

c.  $\text{Area} = \int_{-\epsilon}^{\epsilon} \frac{1}{\epsilon} (1 - |t|/\epsilon) dt = \int_{-1}^1 \Lambda(t) dt = 1$ . As  $\epsilon \rightarrow 0$ , the function becomes narrower and higher, so it approximates a delta function in the limit.

**Problem 2.6**

a. Make use of the formula  $\delta(at) = \frac{1}{|a|} \delta(t)$  to write  $\delta(2t - 5) = \delta[2(t - 5/2)] = \frac{1}{2} \delta(t - \frac{5}{2})$  and use the sifting property of the  $\delta$ -function to get

$$I_a = \frac{1}{2} \left(\frac{5}{2}\right)^2 + \frac{1}{2} \exp\left[-2\left(\frac{5}{2}\right)\right] = \frac{25}{8} + \frac{1}{2} \exp[-5] = 3.1284$$

b. Impulses at  $-10, -5, 0, 5, 10$  are included in the integral. Use the sifting property after writing the expression as the sum of five integrals to get

$$I_b = (-10)^2 + 1 + (-5)^2 + 1 + 0^2 + 1 + 5^2 + 1 + 10^2 + 1 = 255$$

c. Matching coefficients of like derivatives of  $\delta$ -functions on either side of the equation gives  $A = 5, B = 10$ , and  $C = 3$ .

d. Use  $\delta(at) = \frac{1}{|a|} \delta(t)$  to write  $\delta(4t + 3) = \frac{1}{4} \delta(t + \frac{3}{4})$ . The integral then becomes  $I = \frac{1}{4} [e^{-4\pi(-3/4)} + \tan(10\pi \times (-\frac{3}{4}))] = \frac{1}{4} [e^{3\pi} + \tan(-7.5\pi)] = -9.277 \times 10^{13}$ .

e. Use property 5 of the unit impulse function to get

$$\begin{aligned} I_e &= (-1)^2 \frac{d^2}{dt^2} [\cos 5\pi t + e^{-3t}]_{t=2} = \frac{d}{dt} [-5\pi \sin 5\pi t - 3e^{-3t}]_{t=2} \\ &= \left[-(5\pi)^2 \cos 5\pi t + 9e^{-3t}\right]_{t=2} = -(5\pi)^2 \cos 10\pi + 9e^{-6} = -246.73 \end{aligned}$$

**Problem 2.7**

(a), (c), and (e) are periodic. Their periods are 2 s (fundamental frequency of 0.5 Hz), 2 s, and 3 s, respectively. The waveform of part (c) is a periodic train of triangles, each 2 units wide, extending from  $-\infty$  to  $\infty$  spaced by 2 s ((b) is similar except that it is zero for  $t < -1$  thus making it aperiodic). Waveform (d) is aperiodic because the frequencies of its two components are incommensurable. The waveform of part (e) is a doubly-infinite train of square pulses, each of which is one unit high and one unit wide, centered at  $\dots, -6, -3, 0, 3, 6, \dots$ . Waveform (f) is identical to (e) for  $t \geq -1/2$  but 0 for  $t < -1/2$  thereby making it aperiodic.

**Problem 2.8**

a. The result is

$$x(t) = \cos(6\pi t) + 2 \cos(10\pi t - \pi/2) = \operatorname{Re}(e^{j6\pi t}) + \operatorname{Re}(2e^{j(10\pi t - \pi/2)}) = \operatorname{Re}[e^{j6\pi t} + 2e^{j(10\pi t - \pi/2)}]$$

b. The result is

$$x(t) = e^{-j(10\pi t - \pi/2)} + \frac{1}{2}e^{-j6\pi t} + \frac{1}{2}e^{j6\pi t} + e^{j(10\pi t - \pi/2)}$$

c. The single-sided amplitude spectrum consists of lines of height 1 and 2 at frequencies of 3 and 5 Hz, respectively. The single-sided phase spectrum consists of a line of height  $-\pi/2$  at frequency 5 Hz. The double-sided amplitude spectrum consists of lines of height 1, 1/2, 1/2, and 1 at frequencies of  $-5, -3, 3,$  and 5 Hz, respectively. The double-sided phase spectrum consists of lines of height  $\pi/2$  and  $-\pi/2$  at frequencies of  $-5$  and 5 Hz, respectively.

**Problem 2.9**

a. Power. Since it is a periodic signal, we obtain

$$P_1 = \frac{1}{T_0} \int_0^{T_0} 4 \cos^2(4\pi t + 2\pi/3) dt = \frac{1}{T_0} \int_0^{T_0} 2 [1 + \cos(8\pi t + 4\pi/3)] dt = 2 \text{ W}$$

where  $T_0 = 1/2$  s is the period. The cosine in the above integral integrates to zero because the interval of integration is two periods.

b. Energy. The energy is

$$E_2 = \int_{-\infty}^{\infty} e^{-2\alpha t} u^2(t) dt = \int_0^{\infty} e^{-2\alpha t} dt = \frac{1}{2\alpha} \text{ J}$$

c. Energy. The energy is

$$E_3 = \int_{-\infty}^{\infty} e^{2\alpha t} u^2(-t) dt = \int_{-\infty}^0 e^{2\alpha t} dt = \frac{1}{2\alpha} \text{ J}$$

d. Energy. The energy is

$$\begin{aligned} E_4 &= \lim_{T \rightarrow \infty} \int_{-T}^T \frac{dt}{(\alpha^2 + t^2)} = \lim_{T \rightarrow \infty} \frac{1}{\alpha^2} \int_{-T}^T \frac{dt}{\left(1 + (t/\alpha)^2\right)} \\ &= \lim_{T \rightarrow \infty} \frac{1}{\alpha} \tan^{-1} \left[ \frac{t}{\alpha} \right]_{-T}^T = \lim_{T \rightarrow \infty} \frac{1}{\alpha} \left[ \tan^{-1}(T/\alpha) - \tan^{-1}(-T/\alpha) \right] \\ &= \frac{1}{\alpha} \left[ \frac{\pi}{2} - \left( -\frac{\pi}{2} \right) \right] = \frac{\pi}{\alpha} \text{ J} \end{aligned}$$

e. Energy. Since it is the sum of  $x_2(t)$  and  $x_3(t)$ , its energy is the sum of the energies of these two signals, or  $E_5 = 1/\alpha$  J.

f. Energy. The energy is

$$\begin{aligned} E_6 &= \lim_{T \rightarrow \infty} \int_{-T}^T \left[ e^{-\alpha t} u(t) - e^{-\alpha(t-1)} u(t-1) \right]^2 dt \\ &= \lim_{T \rightarrow \infty} \int_{-T}^T \left[ e^{-2\alpha t} u^2(t) - e^{-\alpha t} e^{-\alpha(t-1)} u(t) u(t-1) + e^{-2\alpha(t-1)} u^2(t-1) \right] dt \\ &= \lim_{T \rightarrow \infty} \left\{ \int_0^T e^{-2\alpha t} dt - e^{-\alpha} \int_1^T e^{-2\alpha(t-1)} dt + \int_1^T e^{-2\alpha(t-1)} dt \right\} \\ &= \lim_{T \rightarrow \infty} \left\{ \int_0^T e^{-2\alpha t} dt - e^{-\alpha} \int_0^{T-1} e^{-2\alpha t'} dt' + \int_0^{T-1} e^{-2\alpha t'} dt' \right\} \\ &= \lim_{T \rightarrow \infty} \left\{ -\frac{e^{-2\alpha t}}{2\alpha} \Big|_0^T + e^{-\alpha} \frac{e^{-2\alpha t'}}{2\alpha} \Big|_0^{T-1} - \frac{e^{-2\alpha t'}}{2\alpha} \Big|_0^{T-1} \right\} \\ &= \frac{1}{2\alpha} - \frac{e^{-\alpha}}{2\alpha} + \frac{1}{2\alpha} = \frac{1}{\alpha} \left( 1 - \frac{1}{2} e^{-\alpha} \right) \text{ J} \end{aligned}$$

**Problem 2.10**

a. Power. Since the signal is periodic with period  $2\pi/\omega$ , we have

$$P = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} A^2 |\sin(\omega t + \theta)|^2 dt = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \frac{A^2}{2} \{1 - \cos[2(\omega t + \theta)]\} dt = \frac{A^2}{2} \text{ W}$$

b. Neither. The energy calculation gives

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T \frac{(A\tau)^2 dt}{\sqrt{\tau + jt}\sqrt{\tau - jt}} dt = \lim_{T \rightarrow \infty} \int_{-T}^T \frac{(A\tau)^2 dt}{\sqrt{\tau^2 + t^2}} dt \rightarrow \infty$$

The power calculation gives

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \frac{(A\tau)^2 dt}{\sqrt{\tau^2 + t^2}} dt = \lim_{T \rightarrow \infty} \frac{(A\tau)^2}{2T} \ln \left( \frac{1 + \sqrt{1 + T^2/\tau^2}}{-1 + \sqrt{1 + T^2/\tau^2}} \right) = 0 \text{ W}$$

c. Energy:

$$E = \int_0^{\infty} A^2 t^2 \exp(-2t/\tau) dt = \frac{1}{8} A^2 \tau \sqrt{\frac{\pi\tau}{2}} \text{ W (use a table of integrals)}$$

d. Energy: This is a "top hat" pulse which is height 2 for  $|t| \leq \tau/2$ , height 1 for  $\tau/2 < |t| \leq \tau$ , and 0 everywhere else. Making use of the even symmetry about  $t = 0$ , the energy is

$$E = 2 \left( \int_0^{\tau/2} 2^2 dt + \int_{\tau/2}^{\tau} 1^2 dt \right) = 5\tau \text{ J}$$

e. Energy. The signal is a "house" two units wide and one unit up to the eaves with a equilateral triangle for a roof. Because of symmetry, the energy calculation need be carried out for positive  $t$  and doubled. The calculation is

$$E = 2 \int_0^1 (2-t)^2 dt = -\frac{2}{3} (2-t)^3 \Big|_0^1 = -\frac{2}{3} + \frac{2 \times 8}{3} = \frac{14}{3} \text{ J}$$

f. Power. Since the two terms are harmonically related, we may add their respective powers and get

$$P = \frac{A^2}{2} + \frac{B^2}{2} \text{ W}$$

**Problem 2.11**

- a. Using the fact that the power contained in a sinusoid is its amplitude squared divided by 2, we get

$$P = \frac{2^2}{2} = 2 \text{ W}$$

- b. This is a periodic train of "box cars" 3 units high, 2 units wide, and occurring every 4 units (period of 4 seconds). The power calculation is

$$P = \frac{1}{4} \int_{-1}^1 3^2 dt = \frac{3^2 \times 2}{4} = 4.5 \text{ W}$$

- c. This is a train of triangles 1 unit high, 4 s wide, and occurring every 6 s. Using the waveform period centered at 0, the power calculation is

$$P = \frac{1}{6} \int_{-2}^2 \left(1 - \frac{t}{2}\right)^2 dt = -\frac{1}{6} \frac{2}{3} \left(1 - \frac{t}{2}\right)^3 \Big|_0^2 = \frac{2}{9} \text{ W}$$

- d. This is a train of "houses" each of which is 2 s wide, 1 unit high to the eaves, with an isosceles triangle on top for the roof. They are separated by 4 s (the period). Using the even symmetry of each house, the power calculation is

$$P_d = \frac{2}{4} \int_0^1 (2-t)^2 dt = -\frac{1}{2} \frac{(2-t)^3}{3} \Big|_0^1 = -\frac{1}{2} \left(\frac{1}{3} - \frac{2^3}{3}\right) = \frac{7}{6} \text{ W}$$

**Problem 2.12**

- a. The energy is

$$\begin{aligned} E &= \int_0^\infty \left|6e^{(-3+j4\pi)t}\right|^2 dt = 36 \int_0^\infty e^{(-3+j4\pi)t} e^{(-3-j4\pi)t} dt \\ &= 36 \int_0^\infty e^{-6t} dt = -36 \frac{e^{-6t}}{6} \Big|_0^\infty = 6 \text{ J} \end{aligned}$$

The power is 0 W.

- b. This signal is a "top hat" pulse which is 2 for  $2 \leq t \leq 4$ , 1 for  $0 \leq t < 2$  and  $4 < t \leq 6$ , and 0 everywhere else. It is clearly an energy signal with energy

$$E = 2 \times 1^2 + 2 \times 2^2 + 2 \times 1^2 = 12 \text{ J}$$

Its power is 0 W.

c. This is a power signal with power

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T 49e^{j6\pi t} e^{-j6\pi t} u(t) dt = \lim_{T \rightarrow \infty} \frac{49}{2T} \int_0^T dt = \frac{49}{2} = 24.5 \text{ W}$$

Its energy is infinite.

d. This is a periodic signal with power  $P = \frac{2^2}{2} = 2 \text{ W}$ . Its energy is infinite.

e. This is neither an energy nor a power signal. Its energy is infinite and its power is

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T t^2 dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \left. \frac{t^3}{3} \right|_{-T}^T = \lim_{T \rightarrow \infty} \frac{1}{2T} \frac{2T^3}{3} \rightarrow \infty$$

f. This is neither an energy nor a power signal. Its energy is

$$E = \int_1^{\infty} t^{-1} dt = \ln(t) \Big|_1^{\infty} \rightarrow \infty$$

and its power is

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_1^T t^{-1} dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \ln(t) \Big|_1^{\infty} = 0$$

**Problem 2.13**

a. This is a cosine burst from  $t = -6$  to  $t = 6$  seconds. The energy is  $E_1 = \int_{-6}^6 \cos^2(6\pi t) dt = 2 \int_0^6 \left[ \frac{1}{2} + \frac{1}{2} \cos(12\pi t) \right] dt = 6 \text{ J}$

b. The energy is

$$\begin{aligned} E_2 &= \int_{-\infty}^{\infty} \left[ e^{-|t|/3} \right]^2 dt = 2 \int_0^{\infty} e^{-2t/3} dt \text{ (by even symmetry)} \\ &= -2 \left. \frac{e^{-2t/3}}{2/3} \right|_0^{\infty} = 3 \text{ J} \end{aligned}$$

Since the result is finite, this is an energy signal.

c. The energy is

$$E_3 = \int_{-\infty}^{\infty} \{2[u(t) - u(t-8)]\}^2 dt = \int_0^8 4 dt = 32 \text{ J}$$

Since the result is finite, this is an energy signal.

d. Note that

$$r(t) \triangleq \int_{-\infty}^t u(\lambda) d\lambda = \begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases}$$

which is called the unit ramp. Thus the given signal is a triangle between 0 and 20. The energy is

$$E_4 = \int_{-\infty}^{\infty} [r(t) - 2r(t-10) + r(t-20)]^2 dt = 2 \int_0^{10} t^2 dt = \frac{2}{3} t^3 \Big|_0^{10} = \frac{2000}{3} \text{ J}$$

where the last integral follows because the integrand is a symmetrical triangle about  $t = 10$ . Since the result is finite, this is an energy signal.

**Problem 2.14**

a. This is a cosine burst nonzero between 0 and 2 seconds. Its power is 0. Its energy is

$$E_1 = \int_0^2 \cos^2(10\pi t) dt = \frac{1}{2} \int_0^2 [1 + \cos(20\pi t)] dt = 1 \text{ J}$$

b. This is a periodic sequence of triangles of period 3 s. Its energy is infinite. Its power is

$$P_2 = \frac{2}{3} \int_0^2 (1 - t/2)^2 dt = \frac{4}{9} \text{ J}$$

c. This is an energy signal. Its power is 0. Using evenness of the integrand, its energy is

$$\begin{aligned} E_3 &= 2 \int_0^{\infty} e^{-2t} \cos^2(2\pi t) dt = \int_0^{\infty} e^{-2t} dt + \int_0^{\infty} e^{-2t} \cos(4\pi t) dt \\ &= \frac{1}{2} + \frac{2}{4 + 16\pi^2} \text{ J} \end{aligned}$$

d. This is an energy signal. Its energy is

$$E_4 = 2 \int_0^1 (2 - t)^2 dt = -\frac{2}{3} (2 - t)^3 \Big|_0^1 = \frac{14}{3} \text{ J}$$