Chapter 1

1.1 Equation 1.7 is the integrated form of Fourier's Law of Conduction in one dimension for constant thermal conductivity and steady-state operation. If a more accurate model is desired, the variation of thermal conductivity with temperature may be accounted for by using

$$k = k_0(1 + C_1(T - T_o))$$

where k is thermal conductivity, k_0 is thermal conductivity at some reference temperature T_0, C_1 is a constant, and T is temperature. Determine k_0 and C_1 for copper if T_0 = 650 K. Use the graphs in this chapter to obtain the necessary data.

Solution:
$k = k_0(1 + C_1(T - T_o))$ to fit data on copper. At T_0 = 650 K, k_0 = 360 W / (m·K) from Figure 1.3. At 200 K, k_0 = 415 W / (m·K) also from Figure 1.3. Second point is needed and selected arbitrarily. Substitute into the equation:

$$k = k_0(1 + C_1(T - T_o))$$

$$415 = 360\ (1 + C_1(200 - 650))$$

$$C_1 = \frac{(415 / 360 - 1)}{-450} = -3.4 \times 10^{-4}$$ so the equation becomes

$$\boxed{k = 360(1 - 3.4 \times 10^{-4}(T - 650))}$$ [SI units]

1.2 Repeat Problem 1 for glycerin if T_0 = 170°F.

Solution:
$k = k_0(1 + C_1(T - T_o))$ to fit data on glycerin. At T_0 = 170 + 460 = 630°R, k_0 = 0.17 BTU / (hr·ft²·°R). At 590°R, k_0 = 0.18 BTU / (hr·ft²·°R) selected arbitrarily. Both data points from Figure 1.4. Substitute into the equation:

$$k = k_0(1 + C_1(T - T_o))$$

$$0.18 = 0.17\ (1 + C_1(590 - 630))$$

$$C_1 = \frac{(0.18 / 0.17 - 1)}{(590 - 630)} = -1.47 \times 10^{-3}$$ so the equation becomes

$$\boxed{k = 0.17(1 - 1.47 \times 10^{-3}(T - 630))}$$ [Engr units]

1.3 Repeat Problem 1 for hydrogen if the reference temperature is 350 K.

Solution:
$k = k_0(1 + C_1(T - T_o))$ to fit data on H_2. At T_0 = 350 K, k_0 = 0.2 W / (m·K) from Figure 1.5. At 600 K, k_0 = 0.315 W / (m·K) also from Figure 1.5. Second point is needed and selected arbitrarily. Substitute into the equation:

$$k = k_0(1 + C_1(T - T_o))$$

$$0.315 = 0.2\,(1 + C_1(600 - 350))$$

$$C_1 = \frac{(0.315 / 0.2 - 1)}{600 - 350} = -2.3 \times 10^{-3}$$ so the equation becomes

$$\boxed{k = 0.2(1 - 2.3 \times 10^{-3}(T - 350))}$$ [SI units]

1.4 Equation 1.7 was obtained by integration of Equation 1.6, assuming a constant thermal conductivity and steady-state operation. If thermal conductivity in Equation 1.6 is not a constant but instead is written as

$$k = k_0(1 + C_1(T - T_o))$$

where k is thermal conductivity, k_0 is thermal conductivity at some reference temperature T_0, C_1 is a constant, and T is temperature, how is Equation 1.7 affected? Derive the new expression.

Solution:

$$\int_0^L \frac{q_x}{A}\,dx = \int_{T_1}^{T_2} -k\,dT \qquad \text{Substitute for } k = k_0(1 + C_1(T - T_o))$$

$$\int_0^L \frac{q_x}{A}\,dx = \int_{T_1}^{T_2} -k_0(1 + C_1(T - T_o))dT = \int_{T_1}^{T_2} (-k_0 - k_0 C_1 T + k_0 C_1 T_o)dT)$$

For constant q_x and A,

$$\frac{q_x}{A} L = -k_0(T_2 - T_1) - k_0 C_1(T_2^2 - T_1^2)/2 + k_0 C_1 T_o(T_2 - T_1)$$

$$\frac{q_x}{A} L = (-k_0 + k_0 C_1 T_o)(T_2 - T_1) - \frac{k_0 C_1}{2}(T_2 - T_1)(T_2 + T_1)$$

$$\frac{q_x}{A} L = [-k_0 + k_0 C_1 T_o - \frac{k_0 C_1}{2}(T_2 + T_1)](T_2 - T_1)$$

$$\frac{q_x}{A} L = -k_0[1 - C_1 T_o + \frac{C_1}{2}(T_2 + T_1)](T_2 - T_1) \qquad \text{or}$$

$$\boxed{q_x = -\frac{k_0 A}{L}(T_2 - T_1)\,[1 - C_1 T_o + \frac{C_1}{2}(T_2 + T_1)]}$$

1.5 A fire wall separates two rooms. In one room, a fire has started. The fire wall is made of stainless steel that is 1/2 in thick. Heat in the room is sufficient to raise the outside surface temperature of one side of the wall to 350°F. If the thermal conductivity of the metal wall is 9.4 BTU/(hr·ft·°R), and the heat flux is 6.3 BTU/(s·ft^2), determine the surface temperature of the other side of the fire wall.

Solution:

$$\frac{q_x}{A} = -\frac{k}{L}(T_2 - T_1)$$ Substituting,

$$(6.3 \text{ BTU/(s·ft}^2))(3600 \text{ s/hr}) = (9.4 \text{ BTU/(hr·ft·°R)})\frac{(350°\text{F} - T)}{(0.5/12) \text{ ft}}$$

Solving,

$$\boxed{T = 249.5°\text{F}}$$

1.6 Figure P1.6 shows a cross section of two materials. Also shown is a coordinate system with a graph of the steady-state temperature vs. distance profile. Based on T vs. x data, which material (A or B) has the higher thermal conductivity? Prove it.

Solution:
Material on left, subscript L; on right, subscript R

$$\left.\frac{\Delta T}{\Delta x}\right|_L = \left.\frac{\Delta T}{\Delta x}\right|_R$$ Line on the right is "steeper"

Same heat being conducted through both materials,

$$\frac{q_x}{A} = k_R \left.\frac{\Delta T}{\Delta x}\right|_R = k_L \left.\frac{\Delta T}{\Delta x}\right|_L$$ With $\left.\frac{\Delta T}{\Delta x}\right|_L < \left.\frac{\Delta T}{\Delta x}\right|_R$ then

$$\boxed{k_R < k_L}$$

1.7 The floor of a brick fireplace is to be appropriately insulated. It is proposed to use a 1/2–in thick sheet of asbestos. The upper-surface temperature of the asbestos can reach 1000°F while the underside temperature is only 400°F. How much heat per unit area is conducted through the asbestos?

Solution:
Asbestos, App Table B-3; $k = 0.066$ BTU/(hr·ft·°R) L = 0.5 in = 0.417 ft

$$\frac{q_x}{A} = -\frac{k}{L}(T_2 - T_1)$$ Substituting,

$$\frac{q_x}{A} = (0.066 \text{ BTU/(hr·ft·°R)})\frac{(1000°\text{F} - 400°\text{F})}{(0.0417)\text{ ft}}$$

Solving,

$$\boxed{\frac{q_x}{A} = 949 \text{ BTU/(hr·ft}^2\text{·°F)}}$$

1.8 Repeat Problem 7 for expanded cork board.

Solution:
Expanded Cork, App Table B-3; $k = 0.021$ BTU/(hr·ft·°R)
L = 0.5 in = 0.0417 ft

$$\frac{q_x}{A} = -\frac{k}{L}(T_2 - T_1)$$ Substituting,

$$\frac{q_x}{A} = (0.021 \text{ BTU/(hr·ft·°R)})\frac{(1000°\text{F} - 400°\text{F})}{(0.0417)\text{ ft}}$$

Solving,

$$\boxed{\frac{q_x}{A} = 302.2 \text{ BTU/(hr·ft}^2\text{·°F)}}$$

1.9 The guarded-hot-plate apparatus described in Example 1.1 is used to measure thermal conductivity of cork. By mistake, rather than obtaining two samples of the same cork, one sample each of a cork board and of expanded cork are used. A 1-in thick piece of expanded cork is placed in the top position, and a 1-in thick piece of cork board is placed in the lower position. The input to the main heater is 120 V with a current of 0.05 A. The temperature of surface 1 is 90°F, and the temperature at surface 2 is 70°F. Using values of thermal conductivity in Appendix Table B.3, determine the expected temperature reading at surface 3. Take the effective area to be 3 ft^2.

Solution:

Expanded Cork, App Table B-3; $k_e = 0.021$ BTU/(hr·ft·°R)

Cork Board $k_b = 0.024$ BTU/(hr·ft·°R)

$A = 3$ ft^2 $L = 1/12$ ft $= 0.0833$ ft

Total heat in is $q = 120$ V(0.05 A) = 6 W. Converting, from A-2,

$$q = \frac{6}{2.9288 \text{ x } 10^{-1}} = 20.5 \text{ BTU/hr}$$

This is the total heat passing through both samples. So

$$\frac{q}{A} = +\frac{k_e}{L_e}(T_1 - T_2) + \frac{k_b}{L_b}(T_1 - T_3)$$ Substituting,

$$\frac{20.5}{3} = \frac{0.021}{0.0833}(90 - 70) + \frac{0.024}{0.0833}(90 - T_3)$$

$6.833 = 5.04 + 0.294(90 - T_3)$ $(90 - T_3) = 6.099$ or

$T_3 = 83.9$°F

1.10 A guarded-hot-plate heater (described in Example 1.1) is used to measure thermal conductivity. A 2-cm thick sheet of Plexiglas® is placed in the top position, and a 4-cm thick sheet of Plexiglas is placed in the lower position. The power input is 120 V; A = 0.813 amps, and the temperature at surface 1 is 35°C. What is the expected temperature at surface 3 if T_2 = 30°C? Take the area to be 1 m^2.

Solution:

Plexiglas, App Table B-3; $k_e = 0.195$ W/(m·K) = constant both materials

$A = 1\ m^2$ $\quad L_{12} = 0.02$ m $\quad L_{13} = 0.04$ m

$T_1 = 35°C$ $\quad T_2 = 30°C$

Total heat in is q = 120 V(0.813 A) = 97.56 W. This is the total heat passing through both samples. So

$$\frac{q}{k\,A} = +\frac{(T_1 - T_2)}{L_{12}} + \frac{(T_1 - T_3)}{L_{13}}$$ Substituting,

$$\frac{97.56\ \text{W}}{(0.195\ \text{W}/(\text{m·K}))(1\ \text{m}^2)} = \frac{35-30}{0.02} + \frac{35-T_3}{0.04} \qquad [°\text{C}/\text{m}]$$

$$500.3 = 250 + \frac{35-T_3}{0.04}$$ Solving,

$$\boxed{T_3 = 25°\text{C}}$$

1.11 Thermal conductivity of insulating materials can be measured with a guarded-hot-plate heater (see Example 1.1). Suppose a 1/4-in thick piece of linoleum is placed in the top position and kapok in the lower position. The main heater operates at 120 V and draws 2 A. The temperature at surface 1 is 100°F. Determine the thickness of kapok required for the temperature at surface 3 to equal that at surface 2. Take the temperature at surface 2 to be 50°F and the area to be 1.3 ft^2.

Solution:

Linoleum, App Table B-3; k_e = 0.047 BTU/(hr·ft·°R) L = 0.25 in = 0.02083 ft

Kapok k_b = 0.02 BTU/(hr·ft·°R) L_k = unknown

A = 3 ft^2 L = 1/12 ft = 0.0833 ft

Total heat in is q = 120 V(2 A) = 240 W. Converting, from A-2,

$$q = \frac{240}{2.9288 \text{ x } 10^{-1}} = 819.4 \text{ BTU/hr}$$

This is the total heat passing through both samples. So

$$\frac{q}{A} = +\frac{k_e}{L_e}(T_1 - T_2) + \frac{k_b}{L_b}(T_1 - T_3)$$ Substituting,

$$\frac{819}{4.1.3} = \frac{0.047}{0.02083}(100 - 50) + \frac{0.02}{L_k}(100 - 50)$$

$$630.5 = 113 + \frac{1}{L_k}$$

$$\boxed{L_k = 0.00193 \text{ ft} = 0.023 \text{ in}}$$

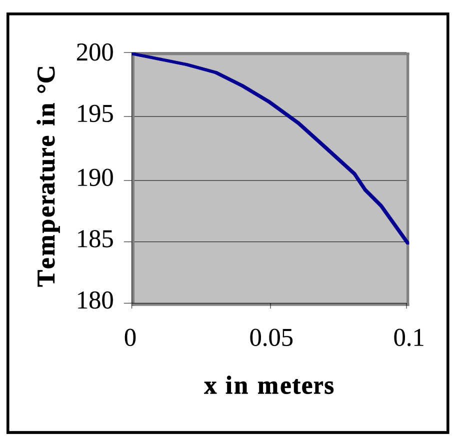
1.12 The temperature distribution in a vertical stainless steel (type 304) wall is given by

$$T = 200 - 1\,500x^2$$

where x is measured from the left face and is expressed in m, and T in °C. The wall is 10 cm thick. Graph the temperature distribution. Is heat being applied to one or both sides of the metal, or is it being allowed to cool?

Solution:

x in m	T in °C
0	200
0.02	199.4
0.04	197.6
0.06	194.6
0.08	190.4
0.1	185



Left face is being cooled after having been heated.

1.13 The apparatus of Figure 1.8 is used to measure thermal conductivity of platinum, located in place of the aluminum. The temperature data of the stainless steel show an average $\Delta T/\Delta x$ of 40°C / cm. Based on the data of Appendix Table B.1, what is the expected $\Delta T/\Delta x$ for the platinum?

Solution:

Platinum App Table B-1 k_p = 71.4 W / (m·K)

Stainless steel k_{ss} = 14.4 W / (m·K)

$$\frac{q_x}{A} = k\frac{\Delta T}{\Delta x}\bigg|\,\text{ss} = k\frac{\Delta T}{\Delta x}\bigg|\,\text{p}$$

$$\frac{\Delta T}{\Delta x}\bigg|\,\text{ss} = 40°\text{C / m}$$

$$\frac{k_{ss}}{k_p}\frac{\Delta T}{\Delta x}\bigg|\,\text{ss} = \frac{\Delta T}{\Delta x}\bigg|\,\text{p}$$

$$\frac{\Delta T}{\Delta x}\bigg|\,\text{p} = \frac{14.4}{71.4}\frac{40}{1}$$

$$\boxed{\frac{\Delta T}{\Delta x}\bigg|\,\text{p} = 8.07\ °\text{C / m}}$$

1.14 A sweater made of wool is worn to “keep warm” during times of cold weather. Suppose a 1 / 4–in thick wool sweater is worn over a shirt. The temperature at the inside surface of the wool is 94°F while the outside-surface temperature of the wool is 90°F. Determine the heat transfer per unit area through the wool.

Solution:

Wool k = 0.022 BTU / (hr·ft·°R)

$$\frac{q_x}{A} = k\frac{\Delta T}{\Delta x} = 0.022\,\frac{94 - 90}{0.25 / 12}$$

$$\boxed{\frac{q_x}{A} = 4.26\ \text{BTU / (hr·ft}^2\text{)}}$$

1.15 Figure P1.15 is a schematic of two copper rods set up in a one-dimensional heat-flow situation. On the left a copper rod of constant diameter is heated by a hot plate at the bottom and cooled with water at the top. The rod has 10 thermocouples attached, spaced 1 in apart. The rod has a 2-in diameter. The second rod is tapered. At its bottom, the diameter is 1 in, while at its top the diameter is 2 in. The rod is insulated, heated, and instrumented in the same way that the straight rod is. The heat input to both rods is the same, which results in a copper surface temperature at the hot plate of 150°F. At the water chamber of the straight copper rod, the temperature is recorded as 100°F. Construct a graph of temperature vs. distance for both rods. The quantity $\Delta T/\Delta x$ in the straight rod is constant. What quantity is constant in the tapered rod?

Solution:

Copper App Table A-3	k = 233 BTU / (hr·ft·°R)	
Rod on left subscript of L	T_{1L} = 150°F	T_{10L} = 100°F
Rod on right subscript of R	T_{1R} = 150°F	

$$q_L = kA_L \frac{T_{1L} - T_{10L}}{9\Delta x} = kA_L \frac{T_{1L} - T_{2L}}{\Delta x} = kA_L \frac{T_{2L} - T_{3L}}{\Delta x} = \text{etc.}$$

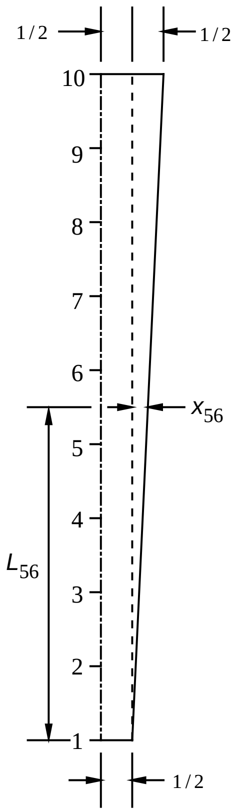
$$q_L = 233 \frac{\pi(2/12)^2}{4} \frac{150 - 100}{9(1/12)} = 338.9 \text{ BTU/hr; constant and equal in both rods.}$$

$$kA_L \frac{T_{1L} - T_{10L}}{9\Delta x} = kA_L \frac{T_{1L} - T_{2L}}{\Delta x}$$ With Δx, k, and A constant, we get

$$\frac{T_{1L} - T_{10L}}{9} = T_{1L} - T_{2L} = T_{2L} - T_{3L} = \text{etc.}$$

Each Δx corresponds to a temperature drop of 5.6°F for the straight rod. So

T_{1L} = 150°F; T_{3L} = 138.9°F; T_{5L} = 127.8°F; T_{7L} = 116.7°F; T_{9L} = 105.6°F; T_{10L} = 100°F.



For the tapered rod, set up the following sketch and use similar triangles to find the area at the midpoint between adjacent thermocouples. Sample calculation to find the diameter at the midpoint between 5 & 6 is as follows:

$$\frac{L_{56}}{x_{56}} = \frac{L_{\text{overall}}}{0.5} = \frac{9}{0.5}\,;\; x_{56} = \frac{0.5L_{56}}{9} = \frac{L_{56}}{18}$$

$$L_{56} = 4.5\text{ in} \qquad x_{56} = \frac{4.5}{18} = 0.25$$

Then $R_{56} = 0.5 + x_{56} = 0.5 + 0.25 = 0.75$ in

$D_{56} = 2R_{56} = 1.5$ in

$$A_{56} = \frac{\pi D_{56}^2}{4(12)^2} = 0.01227\text{ ft}^2$$

As can be seen, the area varies. So in the equation where $T_1 - T_2$ appears for example, we use the average area or area bsed on average diameter between the corresponding diameters D_1 and D_2.

A chart of calculations is as follows:

	L inches	x inches	R inches	Area ft^2	T°F	
					150.0	T_1
1–2	0.5	0.028	0.528	0.00608	130.1	T_2
2–3	1.5	0.083	0.583	0.00742	113.7	T_3
3–4	2.5	0.139	0.639	0.00891	100.1	T_4
4–5	3.5	0.194	0.694	0.01052	88.6	T_5
5–6	4.5	0.250	0.750	0.01227	78.7	T_6
6–7	5.5	0.306	0.806	0.01416	70.2	T_7
7—8	6.5	0.361	0.861	0.01618	62.7	T_8
8–9	7.5	0.417	0.917	0.01833	56.1	T_9
9–10	8.5	0.472	0.972	0.02062	50.2	T_{10}

Now

$$q_R = k_R A_R \frac{\Delta T}{\Delta x} = kA_{12}\frac{T_1 - T_2}{x_2 - x_1} \;=\; kA_{23}\frac{T_2 - T_3}{x_3 - x_2} \quad \text{etc.}$$

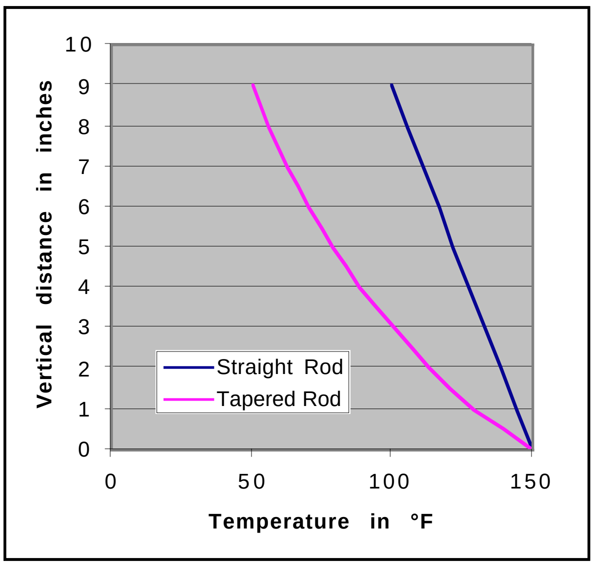
For copper, k = 233 BTU / (hr·ft·°R), and the Δx values are 1 inch = 0.0833 ft. Also q_R = 338.9 BTU / hr found earlier. So that

$$\frac{q_R(x_2 - x_1)}{k} = \frac{338.9(0.0833)}{233} = 0.121 = \text{a constant. Substituting,}$$

$$0.121 = A_{12}(T_1 - T_2); \qquad T_1 = 150°\text{F}; \qquad T_2 = T_1 - \frac{0.121}{0.00608} = 130.1°\text{F}$$

In general, $\quad T_i = \dfrac{0.121}{A_{ij}} = T_j$

Calculations made with this equation are on previous chart. Graph follows.



1.16 It is desired to test the effectiveness of a viscous liquid lubricant that is to be used in sealed bearings. One of the tests to be conducted is to heat a ball bearing to 200°F and submerge the bearing in the lubricant, which is to be kept at 100°F. Determine the convective heat-transfer rate when the bearing is first submerged. Take the convective heat-transfer coefficient between bearing and lubricant to be 100 BTU / hr·ft2·°R. The bearing diameter is 0.4375 in.

Solution:

$A = 4\pi R^2$ for a sphere; D = 0.4375 in = 0.03646 ft R = 0.01823 ft

$A = 4\pi(0.01823)^2 = 0.00418\ \text{ft}^2$

$q = \bar{h}_c A(T_w - T_\infty) = [150\ \text{BTU}/(\text{hr}\cdot\text{ft}^2\cdot{}^\circ\text{R})](0.00418\ \text{ft}^2)(200 - 100)^\circ\text{R}$

$q = 62.7\ \text{BTU}/\text{hr}$

1.17 The side of a building convects heat away to the air in contact with it. The building's outside-wall temperature is 25°C, and the surrounding air is at 15°C. The wall is 3 m tall by 18 m wide. Estimate the heat convected away.

Solution:

$\bar{h}_c$ is estimated because it was not given. From Table 1.2, we select 5 W / (m^2·K).

$q = \bar{h}_c A(T_s - T_\infty) = (5)(50)(10)$

$q = 2\,700\ \text{W} = 2.7\ \text{kW}$

If 25 W / (m^2·K) is selected instead, q = 13.5 kW. Sketch:

1.18 A bathtub is filled to a depth of 14 in with water whose temperature is 130°F. The side walls, front, and back of the tub have a surface temperature of 72°F, and they receive heat from the water. The tub sides can be approximated as vertical walls being 4 ft long. The front and back are 30 in wide. Estimate the heat convected to the tub walls. Sketch the temperature profile in the vicinity of a wall.

Solution:

T_w = 72°F; T_∞ = 130°F

Area of one side = 4 x 1 = 4 ft^2; Area of back or front = (30 / 12) x 1 = 2.5 ft^2

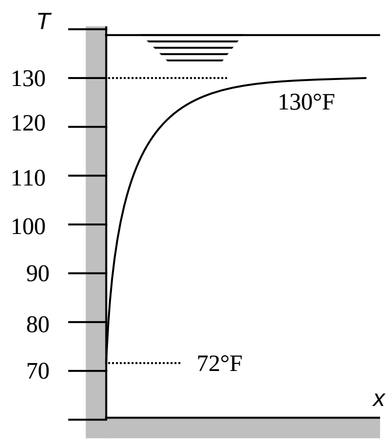
Total area = A = 4 x 2 + 2.5 x 2 = 13 ft^2

Table 1.2, $3.5 \le \bar{h}_c \le 17$ BTU / (hr·ft^2·°R)

$q = \bar{h}_c A(T_s - T_\infty) = (\bar{h}_c)(13)(72 - 130) = -754\,\bar{h}_c$

So $-2{,}639 \le q \le -12{,}818$ BTU

where the negative sign indicates heat transfer from the fluid to the wall. Note that Table 1.2 values may not apply, but at this point, that's all that is available.



1.19 The temperature profile in a brick masonry wall is steady and linear, as shown in Figure P1.19. At surface 1, the temperature is 30°C. At surface 2, the temperature is 20°C. On the left of the wall is water at a bulk temperature of 38°C. Determine the convective heat-transfer coefficient between the water and the wall. Take the area to be 1 m^2. Determine the heat transferred through the wall.

Solution:

Brick masonry $k = 0.658$ W/(m·K) $A = 1$ m^2 $\Delta T = 30 - 20 = 10$°C

$\Delta x = 10$ cm $= 0.01$ m $T_w = 30$°C $T_\infty = 38$°C

Heat in by conduction $= q = -kA\dfrac{\Delta T}{\Delta x} = 0.658(1)\dfrac{10}{0.03} = 21.9$ W

Heat conducted = Heat convected

$q = \bar{h}_c A(T_w - T_\infty)$ $\quad$ $\bar{h}_c = \dfrac{-q}{A(T_w - T_\infty)} = \dfrac{-21.9}{1(30-38)}$

$\boxed{\bar{h}_c = 2.74 \text{ W/(m}^2\cdot\text{K)}}$

1.20 A hydraulic elevator operates by a pump that moves hydraulic oil into or out of a telescoping cylinder that supports the elevator car. The oil is stored in a tank near the pump. When the elevator is not used much, such as during evenings, and when the ambient temperature is low, the oil loses more heat than is desired. The oil density decreases with temperature, and in the morning the elevator floor is not level with the hall floor. The elevator drops about 8–10 cm, which is unacceptable, and is due to the oil being cold. So several electric immersion heaters are used to maintain the oil temperature at 25°C. The tank is shown in Figure P1.23. The tank walls are thin enough so that the temperature within the wall is uniform and also at 25°C. The ambient air temperature is 15°C. Estimate the heat lost by convection through the four vertical tank walls. Use an average value for the film coefficient.

Solution:

$T_w = 25$°C $T_\infty = 15$°C Area of front & back = 2(1.3)(0.8) = 2.08 m^2

Area of sides = 2(1.1)(0.8) = 1.76 m^2 From Table 1.2, the convection coefficient varies from 5 to 25 W/(m^2·K); the average is 15 W/(m^2·K).

$q = \bar{h}_c A(T_w - T_\infty) = 15(2.08 + 1.76)(25 - 15)$

$\boxed{q = 576 \text{ W}}$

1.21 The data presented in Example 1.4 in conjunction with the apparatus of Figure 1.11 allowed for finding a convective heat-transfer coefficient. The film coefficient calculated is the one defined in Newton's Law of Cooling. Suppose that we use a different definition for a convective heat-transfer coefficient. Specifically, where T_{win} is the wall temperature at the inlet, T_{w2} is the wall temperature at location 2, T_{bin} is the bulk fluid temperature at the inlet, and T_{b2} is the bulk fluid temperature at location 2. With the data of Example 1.4, determine the film coefficient $\overline{h}_a$. Is there any advantage of using this definition over the alternative?

Solution:

$\dot{m} = 0.5$ kg / s $\quad c_p = 4.2 \times 10^3$ J / (kg·K) $\quad T_{win} = T_w = T_{w2} = 75°C$ = constant wall temp
$T_{b2} = 21.4°C \quad T_{bin} = 20°C \quad D = 0.024\,3$ m $\quad L = 0.2$ m

$$q = \overline{h}_a A \frac{(T_{win} - T_{bin}) + (T_{w2} - T_{b2})}{2}$$

$$\dot{m}c_p(T_{b2} - T_{bin}) = \overline{h}_a A \frac{(T_{win} - T_{bin}) + (T_{w2} - T_{b2})}{2}$$

Solving,

$$\overline{h}_a = \frac{2\dot{m}c_p(T_{b2} - T_{bin})}{\pi DL[(T_{win} - T_{bin}) + (T_{w2} - T_{b2})]}$$

$$\overline{h}_a = \frac{2(0.5)(4\,200)(21.4 - 20)}{\pi(0.0243)(0.2)[75 - 20 + 75 - 21.4]}$$

$$\boxed{\overline{h}_a = 3\,546 \text{ W / (m}^2\text{·K)}}$$

This coefficient does not require more temperatures to be measued but is more cumbersome to use than the other.

1.22 Referring to Example 1.4 and Figure 1.11, determine $\bar{h}_c$ for each thermocouple location, given the following data:

T_{b1} = 20.8°C T_{b2} = 22.0°C
T_{b3} = 21.4°C T_{b4} = 22.5°C

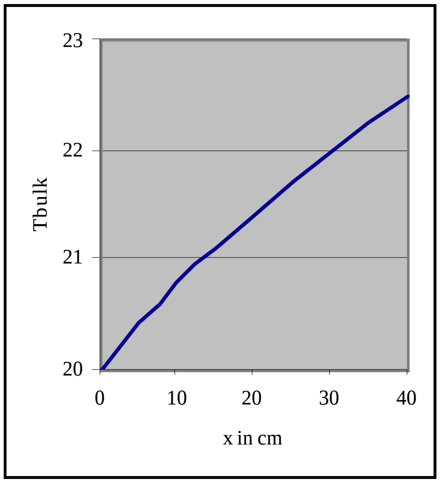
Graph the bulk fluid temperature and the film coefficient vs. distance.

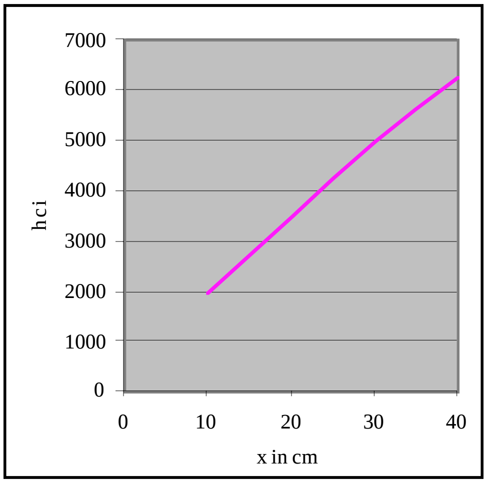
Solution:
From the example,

$\dot{m}$ = 0.5 kg / s c_p = 4.2 x 10^3 J / (kg·K) T_{bin} = 20°C D = 0.024 3 m L = 0.2 m
T_w = 75°C T_{bi} = 21.4°C i = 1, 2, 3, or 4 locations

$$\bar{h}_c = \frac{\dot{m}c_p(T_{bi} - T_{bin})}{\pi DL(T_w - T_{bin})} = \frac{0.5(4\ 200)}{\pi(0.0243)(0.2)}\ \frac{T_{bi} - 20}{75 - 20}$$

$$\bar{h}_c = 2\ 500.7(T_{bi} - 20)$$



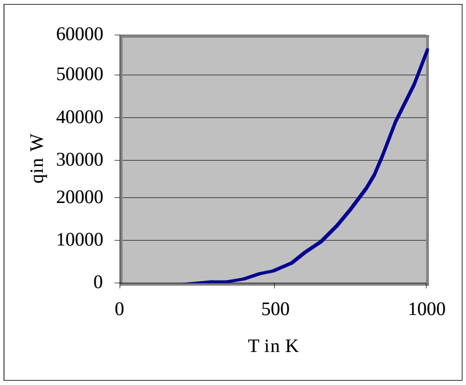


1.23 A black body having a surface area of 1 m^2 radiates heat to the surroundings, which are at 0 K. Determine the rate of heat transfer by radiation from the black body if its temperature is (a) 0 K, (b) 200 K, (c) 400 K, (d) 600 K, (e) 800 K, (f) 1 000 K. Construct a graph of q_r vs. T.

Solution:

$\varepsilon = F_{1\text{-}2} = 1$ $A = 1\ m^2$ $T_2 = 0$ K

$q = \sigma A F_{1\text{-}2}\varepsilon(T_1^4 - T_2^4) = 5.67 \times 10^{-8}(1)(1)(1)(T_1^4 - 0)$ W



T_1 K	q W
0	0
200	90
400	1451
600	7348
800	23224
1000	56700

1.24 A tungsten filament at 5900°R radiates heat in all directions. Assuming the heat receiver is a black body, how much heat per unit area is radiated from the filament? The receiver temperature is 0°R.

Solution:

Tungsten filament Table E-1: $\varepsilon = 0.39$

$\varepsilon = F_{1\text{-}2} = 1$ $A = 1\ m^2$ $T_1 = 5900$°R $T_2 = 0$°R

$$\frac{q}{A} = \sigma F_{1\text{-}2}\varepsilon(T_1^4 - T_2^4) = 0.1714 \times 10^{-8}(1)(0.39)(5900^4 - 0)$$

$$\boxed{\frac{q}{A} = 8.1 \times 10^5\ \text{BTU}/(\text{ft}^2\cdot\text{hr})}$$

1.25 An asphalt driveway is 14 ft wide and 30 ft long. During daylight hours the driveway receives enough energy to raise its temperature to 120°F. Determine the instantaneous heat loss rate into space by radiation from the upper surface of the driveway during nighttime, assuming the surface to be at 120°F and that the surface has an emissivity of 0.9. Neglect convection losses. The geometry factor is 1.

Solution:

$T_1 = 120 + 460 = 580$°R

$q = \sigma A F_{1\text{-}2}\varepsilon(T_1^4 - T_2^4) = 0.1714 \times 10^{-8}(14)(30)(0.9)(580^4 - 0)$

$$\boxed{q = 73{,}320\ \text{BTU}/\text{hr}}$$

1.26 A satellite in orbit uses solar cells to convert sunlight to power. A preliminary analysis determines that the plate will radiate 14 000 W of heat into space when not facing the sun. If the plate is a 6 x 6 ft square, determine the surface temperature of the plate. Assume $\varepsilon = 1$.

Solution:

$A = 6(6) = 36\ \text{ft}^2$; $F_{1\text{-}2} = 1$ $\varepsilon = 1$

$q_r = 14\,000\ \text{W} = 4.78 \times 10^4\ \text{BTU/hr}$ $T_2 = 0°\text{R}$

$$\frac{q}{A} = \sigma F_{1\text{-}2}\varepsilon_1(T_1^4 - T_2^4)$$

$$4.78 \times 10^4 = 0.1714 \times 10^{-8}(36)(T_1^4 - 0^4)$$

$$T_1^4 = 7.74 \times 10^{11}$$

$$\boxed{T_1 = 938°\text{R}}$$

1.27 There are various methods for evaluating shape factor. Consider the sketch of Figure P1.27. The enclosure consists of three planes that are infinite in length (into the page). In this problem we will determine the shape factor for various combinations of surfaces. Using one particular method of evaluating shape factor, it is determined that $F_{1\text{-}2} = \frac{1}{2L_1}(L_2 + L_1 - L_3)$, which is written assuming unit depth into the page. Recall that shape factor is defined as the fraction of total energy leaving one surface that impinges directly upon another surface—F_{1-2} is the fraction of total energy leaving surface 1 that arrives directly on surface 2. There exist various algebraic relations regarding shape factors. One such relationship deals with reciprocity. For example, we can write $A_iF_{i\text{-}j} = A_jF_{j\text{-}1}$

(a) Given the definition of shape factor, what is F_{1-1}, F_{1-2} and F_{1-3}?
(b) Can surface 1 "see itself"? What is the numerical value of F_{1-1}?
(c) Evaluate F_{1-2} and use the results of part (a) to evaluate F_{1-3}.
(d) Evaluate F_{2-1} using reciprocity.
(e) Evaluate F_{3-1} using reciprocity.
(f) Determine F_{2-3} and F_{3-2}.

Leave answers in fractional form.

Solution:

(a) $F_{1\text{-}1} + F_{1\text{-}2} + F_{1\text{-}3} = 1$

(b) $F_{1\text{-}1} = 0$

(c) $F_{1\text{-}2} = \frac{1}{2L_1}(L_2 + L_1 - L_3) = \frac{1}{2(3)}(4 + 3 - 5)$

$\boxed{F_{1\text{-}2} = \frac{1}{3}}$ $F_{1\text{-}1} = 0;$ So $F_{1\text{-}3} = 1 - F_{1\text{-}2} = 1 - 1/3$ or

$\boxed{F_{1\text{-}3} = \frac{2}{3}}$

(d) $A_1F_{1\text{-}2} = A_2F_{2\text{-}1};$ $F_{2\text{-}1} = \frac{A_1}{A_2}F_{1\text{-}2} = \frac{L_1}{L_2}F_{1\text{-}2};$ $F_{2\text{-}1} = \frac{3}{4}\frac{1}{3}$ or

$\boxed{F_{2\text{-}1} = \frac{1}{4}}$

(e) $A_1F_{1\text{-}3} = A_3F_{3\text{-}1};$ $F_{3\text{-}1} = \frac{A_1}{A_3}F_{1\text{-}3} = \frac{L_1}{L_3}F_{1\text{-}3};$ $F_{3\text{-}1} = \frac{3}{5}\frac{2}{3}$ or

$\boxed{F_{2\text{-}1} = \frac{2}{5}}$

(f) $F_{2\text{-}1} + F_{2\text{-}2} + F_{2\text{-}3} = 1;$ $F_{2\text{-}2} = 0$ So $F_{2\text{-}3} = 1 - F_{2\text{-}1} = 1 - 1/4$ or

$\boxed{F_{2\text{-}3} = \frac{3}{4}}$

$F_{3\text{-}1} + F_{3\text{-}2} + F_{3\text{-}3} = 1;$ $F_{3\text{-}3} = 0$ So $F_{3\text{-}2} = 1 - F_{3\text{-}1} = 1 - 2/5$ or

$\boxed{F_{3\text{-}2} = \frac{3}{5}}$

1.28 Two infinite black plates at 500°C and 300°C exchange heat by radiation. Determine the net heat flux from the hot plate to the cold plate.

Solution:
$T_1 = 500 + 273 = 773$ K; $T_2 = 300 + 273 = 573$ K

$$\frac{q}{A} = \sigma F_{1\text{-}2}\varepsilon_1(T_1^4 - T_2^4) = 5.67 \times 10^{-8}(1)(1)(773^4 - 573^4)$$

$$\boxed{\frac{q}{A} = 14\,130 \text{ W/m}^2}$$

1.29 A 2-cm thick plate of steel (1C) receives a radiant heat flux of 1 000 W/m^2. The steel is in an environment where convection can be neglected. The surface temperature on the opposite side of the plate is 25°C. Determine the temperature of the surface on which the radiant energy is incident.

Solution:
$k = 43$ W/(m·K) from Table B-2
$q_r = q_x$ Heat radiated to the plate = heat conducted through it.

$$1\,000 \text{ W/m}^2 = \frac{k}{L}(T_1 - T_2) \qquad T_1 = \frac{1\,000L}{k} + T_2 = \frac{1\,000(0.02)}{43} + 25$$

$$\boxed{T_1 = 25.5°\text{C}}$$

1.30 A radiant heat flux of 30 BTU/(hr·ft^2) is absorbed by a surface, which in turn transfers heat away only by convection to surrounding air. For an ambient air temperature of 85°F and a film coefficient of 0.9 BTU/(hr·ft^2·°R), determine the equilibrium temperature of the surface.

Solution:
q in by radiation = q transferred by convection = 30 BTU/(hr·ft^2) = $h_c(T_w - T_\infty)$

$30 = 0.9(T_w - 85)$ or

$$\boxed{T_w = 118°\text{F}}$$

1.31 A new design of an electric water heater for a home is being analyzed. The heater is to consist of a length of tubing wrapped with an electrical conducting tape, which in turn is well insulated. When electric current passes through the tape, it heats the tube, which heats the water inside. The system is set up such that heating occurs only when water flows through the tube. The inside diameter of the tube is 20 mm, and the tube length is 3 m. The convection coefficient between the tube and water is 1 000 W/(m^2·K), which is an average over the 3-m length, based on an effective temperature difference of 340 K (assumed constant).

(a) Determine the heat transferred to the water, and the downstream temperature of the water if the average water velocity is 1.2 m/s and its density is 1 000 kg/m3.
(b) If the voltage applied to the heater is 120 V, will a 15-A fuse be sufficient in the electrical line? Take the initial water temperature to be 20°C, and the specific heat of water to be 4.2 kJ/(kg·K).

Solution:
D = 20 mm = 0.02 m; L = 3 m h_c = 1 000 W/(m^2·K) ΔT = 340 K
V = 1.2 m/s ρ = 1 000 kg/m^3 c_p = 4 200 J/(kg·K)

$q = h_c A(T_w - T_\infty) = h_c \pi D L \Delta T$ = 1 000(π)(0.02)(3)(340) = 6.4 x 10^4 W = 64 kW

$q = \dot{m} c_p (T_f - T_i)$ where T_f = final water temperature downstream, and T_i = 20°C = the intial water temperature.

$\dot{m} = \rho A V$ where A = cross sectional area = $\pi D^2/4$; and

$\dot{m}$ = 1 000(π(0.02)^2/4)(1.2) = 0.377 kg/s. Substituting,

6.4 x 10^4 = 0.377(4 200)(T_f – 20); solving,

T_f = 60.5°C 120 V x 15 A = 1 800 W; 15 A is too small.

1.32 One way of using stove-burner heat to heat a kitchen effectively (but costly) is to utilize a hot plate. Consider a lighted gas burner. Heat is transferred from the flame to the surroundings by what mode(s) of heat transfer? Suppose a hot plate consisting of cast iron with a 6-in diameter is placed over the burner. The plate absorbs a significant fraction of the energy from the flame and, in turn, the plate radiates heat in all directions. It is observed that use of the hot plate heats the surrounding room (kitchen) very well compared to the burner alone.

Consider a hot plate over a burner. The flame temperature is 1800°F. The convective heat-transfer coefficient between the rising exhaust gases and the plate is 35 BTU/(hr·ft^2·°R). The plate transfers heat to the surroundings primarily by radiation. Determine the equilibrium temperature of the plate, assuming the surroundings to be at 65°F and to behave as a black body. The plate surface has ε = 0.9.

q transferred by convection = q transferred by radiation

$q = h_c A(T_\infty - T_w) = \sigma A F_{w\text{-}2} \varepsilon_w (T_w^4 - T_2^4)$;   area cancels

k = 35 BTU / (hr·ft^2·°R) T_∞ = 1800°F + 460 = 2260°R T_w = ? ε = 0.9
$F_{w\text{-}2}$ = 1 T_2 = 65 + 460 = 525°R   Substituting,

$35(2260 - T_w) = 0.1714 \times 10^{-8}(0.9)(T_w^4 - 525^4)$

$2.269 \times 10^{10}(2260 - T_w) = T_w^4 - 7.597 \times 10^{10}$

$5.127 \times 10^{13} - 2.269 \times 10^{10} T_w = T_w^4 - 7.597 \times 10^{10}$

$T_w^4 + 2.269 \times 10^{10} T_w = 5.135 \times 10^{13}$ Solving by a trial and error procedure,

$\boxed{T_w = 1800°\text{R} = 1340°\text{F}}$

1.33 The sloping roof of a house receives energy by radiation from the sun. The roof surface reaches a steady uniform temperature of 140°F, and the ambient air temperature is 85°F. For roof dimensions of 30 x 18 ft, and a convection heat-transfer coefficient of 3 BTU / (hr·ft^2·°R), determine the heat transferred by convection to the air.

Solution:

$q = \bar{h}_c A(T_w - T_\infty) = 3(30)(18)(140 - 85)$

$\boxed{q = 89{,}100 \text{ BTU / hr}}$
