13.1

Since hammer is in free fall,

$a = -g = -9.81\ m/s^2$

$x\uparrow$ 3m

$v = \int a\,dt = -gt + v_0$ ($v_0 = 0$ since dropped from rest)

$v = -9.81\ t$

$x = \int v\,dt = -\frac{1}{2}gt^2 + x_0$ ($x_0 = 3m$, $x_f = 0$)

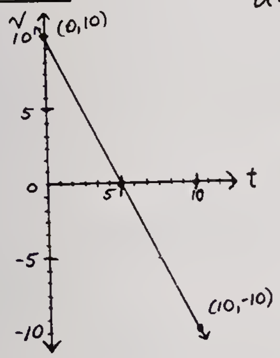
$x = -4.91t^2 + 3 = 0$ when hammer hits pile

Solving $x(t)$ for t_f: $t_f = 0.782\ s$

$v(0.782) = -9.81(0.782)$ $v = -7.67\ m/s$

or $\underline{\underline{v = 7.67\ m/s \downarrow}}$

13.2



a.) Acceleration of particle:

$v = \frac{(-10-10)}{(10-0)}t + 10$

$v = -2t + 10$ [mm/s]

$a = \frac{dv}{dt} \Rightarrow \underline{\underline{a = -2\ mm/s^2}}$

b.) Position of particle:

$x = \int_{t_0}^{t} v\,dt + x_0$ ($x_0 = 0$)

$x = \int_0^t (-2t + 10)\,dt$

$x = -t^2 + 10t \rightarrow$ Solving for $t = 2, 5, 8, 10\,s$:

$\underline{\underline{x(2s) = 16\ [mm]}}$

$\underline{\underline{x(5s) = 25\ [mm]}}$

$\underline{\underline{x(8s) = 16\ [mm]}}$

$\underline{\underline{x(10s) = 0\ [mm]}}$

c) Distance traveled:

$\Delta S = |\Delta X_1| + |\Delta X_2|$

$\underline{\underline{\Delta s(2s) = 16\ [mm]}}$

$\underline{\underline{\Delta s(5s) = 25\ [mm]}}$

$\Delta S(8s) = 25 + |16 - 25| = \underline{\underline{34\ [mm]}}$

$\Delta s(10s) = 25 + |0 - 25| = \underline{\underline{50\ [mm]}}$

13.3

$v = 4 = $ constant, $x = 0$ when $t = 0$

$a = \frac{dv}{dt} \rightarrow \underline{\underline{a(t) = 0}}$

$v = \frac{dx}{dt} = 4 \Rightarrow x = 4t + C = 0$ for $t = 0$

$\therefore C = 0$ $\underline{\underline{x = 4t}}$

13.4

$v = 4t$, $x = 1$ when $t = 1$

$a = \frac{dv}{dt} \Rightarrow \underline{\underline{a(t) = 4}}$ (constant)

$v = \frac{dx}{dt} = 4t \Rightarrow x(t) = 2t^2 + C$

For $x = 1$, $t = 1$ $\therefore$ $1 = 2(1)^2 + C \Rightarrow C = -1$

$\underline{\underline{x(t) = 2t^2 - 1}}$

13.5

$v = \sin 2t$, $x = 0$ when $t = 0$

$a = \frac{dv}{dt} \Rightarrow \underline{\underline{a(t) = 2\cos 2t}}$ (a.)

$v = \frac{dx}{dt} = \sin 2t \Rightarrow x = -\frac{\cos 2t}{2} + C$

$x = 0$ when $t = 0$ $\therefore C = \frac{1}{2}$

$\underline{\underline{x(t) = -\frac{\cos 2t}{2} + \frac{1}{2}}}$ (b.)

By Eq.(b). $-x + \frac{1}{2} = \frac{\cos 2t}{2}$

or $t = (\frac{1}{2})\cos^{-1}(-2x + 1)$ (c.)

Substitution of Eq (c.) into Eq (a.) gives:

$\underline{\underline{a(x) = 2\cos(\frac{1}{2}\cos^{-1}(2 - 4x))}}$

13.6

$v = e^{-2t}$, $x = 2$ when $t = 0$

$a = \frac{dv}{dt} = \underline{\underline{-2e^{-2t}}}$ (a.)

$v = \frac{dx}{dt} = e^{-2t}$ $x = \int v\,dt \Rightarrow x = -\frac{1}{2}e^{-2t} + C$

$x = 2$ when $t = 0$ $\therefore C = \frac{5}{2}$

$\underline{\underline{x(t) = \frac{1}{2}(5 - e^{-2t})}}$ (b.)

Solving Eq (b) for t: $e^{-2t} = 5 - 2x$

$t = -\frac{1}{2}\ln(5 - 2x)$ (c.)

Substitution of Eq (c) into Eq (a) gives:

$a(x) = -2e^{-2[-\frac{1}{2}\ln(5-2x)]} \Rightarrow \underline{\underline{a(x) = 4x - 10}}$

13.7

a.) $\Delta S_f = 2\pi(4000\text{ miles}) = 25132.74$ miles

$\underline{\underline{= 132,700,873\ ft}}$

The displacement of your feet relative to the earth would be zero. $\underline{\underline{\Delta X = 0\,ft}}$

b.) $\Delta S_h = 2\pi[4000\,mi\,(\frac{5280\,ft}{mi}) + 6\,ft]$

$= 132,700,911.4\ ft$

Difference: $132,700,911.4 - 132,700,873.7$

$= 37.7\ ft$

$\therefore$ Your head travels $\underline{\underline{37.7\ ft\ \text{more}}}$

Alternatively: $\underline{\underline{2\pi(6\,ft) = 37.7\ ft}}$

13.8

d - total distance traveled = 2 mi

t - total time over 2 mile stretch

$$t = \frac{1\text{ mile}}{30\text{ mph}} + \frac{1\text{ mile}}{V_{req}}$$

V_{req} = speed required to average 45 mph over 2 mile stretch

$$\frac{d}{t} = 45\text{ mph} \Rightarrow \frac{2\text{mi}}{\frac{1\text{mi}}{30\text{mph}} + \frac{1\text{mi}}{V_{req}}} = 45\text{mph}$$

$$V_{req} = \frac{1\text{mi}}{\frac{2\text{mi}}{45\text{mph}} - \frac{1\text{mi}}{30\text{mph}}} \Rightarrow \underline{\underline{V_{req} = 90\text{ mph}}}$$

13.9

d - total distance traveled = 2 mi

t - total time around 2 mile stretch

$$t = \frac{1\text{mile}}{30\text{ mph}} + \frac{1\text{mile}}{V_{req}}$$

V_{req} = speed required to average 60 mph over 2mile stretch

$$\frac{d}{t} = 60\text{mph} \Rightarrow \frac{2\text{mi}}{\frac{1\text{mi}}{30\text{mph}} + \frac{1\text{mi}}{V_{req}}} = 60\text{ mph}$$

$$V_{req} = \frac{1\text{mi}}{\frac{2\text{mi}}{60\text{mph}} - \frac{1\text{mi}}{30\text{ mph}}} \Rightarrow \underline{\underline{V_{req} = \infty}}$$

(It is impossible to go fast enough to average 60 mph — there is no time left.)

13.10

a.) $v_0 = \left(\frac{1}{6}\frac{\text{mi}}{\text{s}}\right)\left(5280\frac{\text{ft}}{\text{mi}}\right) = 880\text{ ft/s}$

After power is reduced, $a = -g = -32.2\text{ ft/s}^2$

$\therefore v = v_0 + at = 880 - 32.2t = 0$ at highest pt.

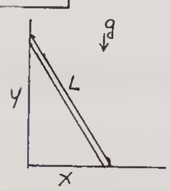
$\therefore t = 880/32.2 \qquad \underline{\underline{t = 27.4\text{ s}}}$

b.) The height the plane rises above the point at which the power is reduced is:

$$x = v_0 t + \tfrac{1}{2}at^2 = 880(27.35) - \tfrac{1}{2}(32.2)(27.35)^2$$

$$x = 12{,}025\text{ ft} \approx \underline{\underline{2.28\text{ mi}}}$$

13.11



$v_y = \dot{y} \qquad a_y = \ddot{y}$

$L^2 = x^2 + y^2$

Differentiating: $0 = \cancel{2}x\dot{x} + \cancel{2}y\dot{y}$

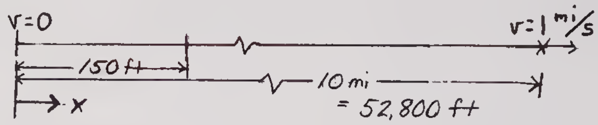
$$\dot{x} = \frac{-y\dot{y}}{x} \Rightarrow \underline{\underline{\dot{x} = \frac{v_y\sqrt{L^2 - x^2}}{x}}}$$

$$\ddot{x} = \frac{-\left(\frac{v_y}{x}\sqrt{L^2-x^2}\right)^2 - v_y^2 + a_y\sqrt{L^2+x^2}}{x}$$

$$\ddot{x} = \frac{a_y\sqrt{L^2-x^2}}{x} - \frac{v_y^2(L^2-x^2)}{x^3} - \frac{v_y^2}{x}$$

$$\underline{\underline{\ddot{x} = \frac{a_y x^2\sqrt{L^2-x^2} - v_y^2 L^2}{x^3}}}$$

13.12



$v = \int a\,dt = at + \cancel{v_0}^{\,0}$ Since $v = 0$ when $t = 0$

$\therefore t = v/a \quad (A)$

$x = \int v\,dt = \tfrac{1}{2}at^2 + \cancel{x_0}^{\,0} = \tfrac{1}{2}at^2$ since $x = 0$ when $t = 0$

$\therefore x = \tfrac{1}{2}at^2 \quad (B)$

By Eqns (A) and (B): $\quad x = \tfrac{1}{2}at^2 = \tfrac{1}{2}\frac{v^2}{a} \quad (C)$

$$\therefore a = \frac{1}{2}\frac{v^2}{x} = \frac{1}{2}\frac{[(1\text{ mi/s})(5280\text{ ft/mi})]^2}{(10\text{ mi})(5280\text{ ft/mi})} = 264\text{ ft/s}^2$$

Substituting $a = 264\text{ ft/s}^2$ into (B):

$150\text{ ft} = \tfrac{1}{2}(264\text{ ft/s}^2)t^2$

$\therefore \underline{\underline{t = 1.066\text{ s}}}$ = time rocket is in contact with guides.

13.13

a.) $a = 32.2\text{ ft/s}^2 \qquad v_0 = 0$

$v = \int a\,dt = at_1 + \cancel{v_0}^{\,0} \Rightarrow v = 32.2t$

$$\left(\frac{5280\text{ ft}}{\text{mi}}\right)\left(186{,}000\frac{\text{mi}}{\text{s}}\right) = 32.2\frac{\text{ft}}{\text{s}^2}(t_1)$$

$t_1 = 30{,}499{,}378.9\text{ s} \Rightarrow \underline{\underline{t_1 = 0.967\text{ years}}}$

b.) At the end of 0.967 years, the distance traveled is:

$$s_1 = \tfrac{1}{2}at_1^2 = \tfrac{1}{2}(32.2)(3.0499\times10^7)^2\left(\tfrac{1}{5280}\right)$$

$$= 2.836\times10^{12}\text{ mi}$$

Let s_2 be distance remaining to be traveled

$\therefore s_2 = 2.5\times10^{13} - s_1 = (2.5 - 0.2836)(10^{13})\text{ mi}$

$= 2.216\times10^{13}\text{ mi}$

Let t_2 = time to travel distance s_2

$$\therefore t_2 = \frac{2.216(10^{13})}{1.86(10^5)} = 1.191(10^8)\text{s} = 3.774\text{ years}$$

$t_{total} = t_1 + t_2 = 0.967 + 3.774 = \underline{\underline{4.74\text{ years}}}$

13.14

a.) $v = 16t - t^2\text{ mi/h} = \frac{1}{60}(16t - t^2)\text{ mi/min}$

$\frac{dv}{dt} = \frac{1}{60}(16 - 2t) = 0$ at $t = 8$ min

$\therefore v_{max} = \frac{1}{60}[16(8) - (8)^2] = \frac{64}{60}\text{ mi/min}$

Hence;

$$S = \frac{1}{60}\int_0^8 (16t - t^2)\,dt + \frac{64}{60}(12-8)$$

$$= \frac{1}{60}\left(8t^2 - \tfrac{1}{3}t^3\right)\Big|_0^8 + \frac{64(4)}{60} \qquad \underline{\underline{S = 9.96\text{ mi}}}$$

b.)

$$v_{ave}(\text{mi/h}) = \frac{9.956\text{ mi}(60\text{min})}{12\text{ min}(1\text{ h})} = \underline{\underline{49.78\text{ mi/h}}}$$

13.15

$a = \frac{dv}{dt} = \frac{k}{v}$

$\therefore \int k\,dt = \int v\,dv \rightarrow \frac{1}{2}v^2 = kt + C$

when $t=0$, $v=2$ $\therefore C = 2$

when $t=3$, $v=4$ $\therefore k=2$

Hence, $v^2 = 4(t+1)$

When $t=8s$, $v^2 = 36$ $\rightarrow$ $\underline{\underline{|v| = 6\ m/s}}$

13.16

$x = C_3t^3 + C_2t^2 + C_1$

a.) $v = \frac{dx}{dt} = \underline{\underline{3C_3t^2 + 2C_2t}}$

$a = \frac{dv}{dt} = \underline{\underline{6C_3t + 2C_2}}$

b.) Distance traveled from $t=t_0$ to $t=t_1$:

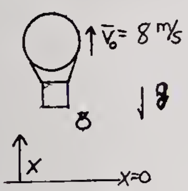
at $t=t_0$ $\quad X_0 = C_3(t_0)^3 + C_2(t_0)^2 + C_1$

at $t=t_1$ $\quad X_1 = C_3(t_1)^3 + C_2(t_1)^2 + C_1$

ΔS = distance traveled $= X_1 - X_0$

$\underline{\underline{\Delta S = C_3(t_1^3 - t_0^3) + C_2(t_1^2 - t_0^2)}}$

13.17



a) $a = -g = -9.81\ m/s^2$

$v = \int a\,dt = -9.81t + v_0$

$v = -9.81t + 8$

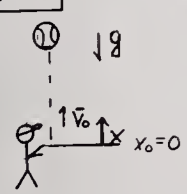
$x = \int v\,dt = -4.91t^2 + 8t + x_0$

$x=0$ when it hits the ground

$0 = -4.91(10)^2 + 8(10) + X_0$ $\rightarrow$ $\underline{\underline{X_0 = 411\ m}}$

b) $v = -9.81(10) + 8$ $\quad v = -90.1\ m/s$ $\quad$ $\underline{\underline{\text{or } 90.1\ m/s \downarrow}}$

13.18



a) $a = -g = -9.81\ m/s^2$

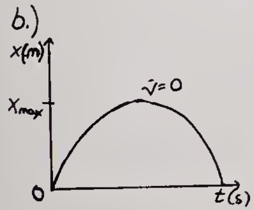
$v = \int a\,dt = -9.81t + v_0$

$x = \int v\,dt = -4.91t^2 + v_0t + \cancel{x_0}^{\,0}$

$0 = -4.91(4)^2 + v_0(4)$ $\quad (x=0,\ t=4s)$

$\therefore$ $\underline{\underline{v_0 = 19.64\ m/s}}$

b.)



$v = -9.81t + 19.64$

As shown on curve, X_{max} occurs when $\bar{v} = 0$.

$\therefore 0 = -9.81t_m + 19.64$

$t_m = 2.00\,s$

$x = -4.91t^2 + 19.64t$

$X_{max} = -4.91(2)^2 + 19.64(2)$

$\underline{\underline{X_{max} = 19.64\ m}}$

13.19

$a = t^3 + 3t$ $[ft/s^2]$ $\quad v=0$ @ $t=0$, $x=0$

a.) acceleration for $t=4s$:

$a = (4)^3 + 3(4)$ $\rightarrow$ $\underline{\underline{a = 76\ ft/s^2}}$

velocity for $t=4s$:

$v = \int_0^t (t^3+3t)\,dt$ $\rightarrow$ $v = \frac{1}{4}t^4 + \frac{3}{2}t^2 + v_0$

Since $v=0$ when $t=0$, $v_0 = 0$

$v(4s) = \frac{1}{4}(4)^4 + \frac{3}{2}(4)^2$ $\rightarrow$ $\underline{\underline{v = 88\ ft/s}}$

displacement for $t=4s$:

$\Delta X = X(4) - X(0) = \int_0^4 v\,dt$

$= \int_0^4 (\frac{1}{4}t^4 + \frac{3}{2}t^2)\,dt$ $= (\frac{1}{20}t^5 + \frac{1}{2}t^3)\Big|_0^4$

$\underline{\underline{\Delta X = 83.2\ ft}}$

b.) $v = \frac{1}{4}t^4 + \frac{3}{2}t^2$

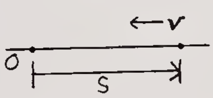
$v = 186{,}000\ mi/s\left(5280\,\frac{ft}{mi}\right) = 9.8208\times10^8\ ft/s$

$9.8208\times10^8 = \frac{1}{4}t^4 + \frac{3}{2}t^2$

Using quadradic equation:

$\underline{t = 250.4\,s}$

13.20



$v = -ks$

a.) $v = \frac{ds}{dt} = -ks$ $\rightarrow$ $\frac{1}{s}\,ds = -k\,dt$

$\int_{S_0}^{S} \frac{1}{s}\,ds = -\int_0^t k\,dt$ $\rightarrow$ $\ln\frac{S}{S_0} = -kt$

$\underline{\underline{S = S_0e^{-kt}}}$

b.) $t=0$, $v=20\ in/s$, $s=10\ in$

Speed $= |v| = ks$

$20 = k(10)$ $\rightarrow$ $\therefore k = 2\,s^{-1}$

$10 = S_0e^{-2(0)}$ $\rightarrow$ $\therefore S_0 = 10\,in$

$S = 10e^{-2t}$ $\rightarrow$ $\therefore t = -\frac{1}{2}\ln\frac{S}{10}$

$t(s=8in) = \underline{\underline{0.112\,s}}$

$t(s=6in) = \underline{\underline{0.255\,s}}$

$t(s=0in) = \underline{\underline{\infty}}$

13.21

$v = t^2 + 20t$ $\rightarrow$ $t = \frac{-20 \pm \sqrt{20^2 - 4(1)(-v)}}{2(1)}$

Negative time does not apply;

$t(v=200) = 7.32s$ $\qquad t(v_2=600) = 16.46\ s$

$\Delta S = \int |v|\,dt = \int v\,dt$ since v is positive

$\Delta S = \int_{7.32}^{16.46} (t^2 + 20t)\,dt$ $= [\frac{1}{3}t^3 + 10t^2]\Big|_{7.32}^{16.46}$

$\underline{\underline{\Delta S = 3530\ ft}}$

13.22

a.) $a = -kv \qquad v = v_0, x = 0$ when $t = 0$

$\int dt = \int \frac{dv}{a(v)} \Rightarrow t - 0 = \int_{v_0}^{v} \frac{dv}{-kv}$

$t = -\frac{1}{k} \ln v \Big|_{v_0}^{v} = -\frac{1}{k} \ln\left(\frac{v}{v_0}\right) \qquad (A)$

$\underline{\underline{v = v_0 e^{-kt}}}$

$\int dx = \int v(t)dt \Rightarrow x - 0 = \int_0^t v_0 e^{-kt} dt$

$x = -\frac{v_0}{k} e^{-kt} \Big|_0^t = -\frac{v_0}{k}\left(e^{-kt} - 1\right)$

$\underline{\underline{x = \frac{v_0}{k}\left(1 - e^{-kt}\right)}} \qquad (B)$

b.) By Eqn (A), $t = -\frac{1}{k}\ln\left(\frac{v}{v_0}\right)$ and,

By Eqn (B), $x = \frac{v_0}{k}\left(1 - e^{-kt}\right)$.

Combining (A) and (B) gives:

$x = \frac{v_0}{k}\left(1 - e^{-k\left(\frac{-1}{k}\ln\frac{v}{v_0}\right)}\right)$ or $\underline{\underline{x(v) = \frac{1}{k}(v_0 - v)}}$

13.23

$a = 4, \quad v = -6, \; x = 0$ when $t = 0$

$v = \int a dt = \int 4 dt \Rightarrow v = 4t + v_0$

$v(t=0) = 0 + v_0 = -6 \quad \therefore v_0 = -6 \qquad \underline{\underline{v = 4t - 6}}$

$x = \int v dt = \int_0^t (4t - 6) dt$

$x = 2t^2 - 6t + x_0 \Rightarrow x(t=0) = 0 - 0 + x_0 = 0 \quad \therefore x_0 = 0$

$\underline{\underline{x = 2t^2 - 6t}}$

From $v(t) \Rightarrow t = \frac{v+6}{4}$

Substitution of t into $x(t)$ gives

$x = 2\left(\frac{v+6}{4}\right)^2 - 6\left(\frac{v+6}{4}\right) = \frac{1}{8}v^2 - 4.5$

$\underline{\underline{v = \sqrt{8x+36} = 2\sqrt{2x+9}}}$

13.24

$a = 4t; \quad v = 2, \; x = 1$ when $t = 1$

$v = \int_{t_1}^{t} a\, dt + v_1 \Rightarrow v = \int_1^t 4t\, dt + v_1$

$v = 2t^2 - 2 + v_1 \Rightarrow v(t=1) = 2 \quad \therefore v_1 = 2$

$\therefore \underline{\underline{v(t) = 2t^2}} \qquad (A)$

$x = \int_{t_1}^{t} v\, dt + x_1 \Rightarrow x = \int_1^t 2t^2 dt + x_1$

$x = \frac{2}{3}(t^3 - 1^3) + x_1 \Rightarrow x(t=1) = 1 = x_1 \quad \therefore x_1 = 1$

$\therefore \underline{\underline{x = \frac{2}{3}t^3 + \frac{1}{3}}} \qquad (B)$

By Eqn (A), $t = \sqrt{v/2} \qquad (C)$

By Eqn (B) and (C), $\quad x = \frac{2}{3}\left(\frac{v}{2}\right)^{3/2} + \frac{1}{3}$

$\therefore \underline{\underline{v = 2\left(\frac{3}{2}x - \frac{1}{2}\right)^{2/3}}}$

13.25

$a = e^{-2t}; \quad v = 0, \; x = 2$ when $t = 0$

$v = \int a\, dt \Rightarrow v = \int e^{-2t} dt$

$v = -\frac{1}{2}e^{-2t} + v_0 \Rightarrow v(0) = 0 = -\frac{1}{2}(1) + v_0$

$\therefore v_0 = \frac{1}{2} \qquad \therefore \underline{\underline{v(t) = \frac{1}{2}\left(1 - e^{-2t}\right)}} \qquad (A)$

$x = \int v\, dt \Rightarrow x = \int\left(\frac{1}{2} - \frac{1}{2}e^{-2t}\right) dt$

$x = \frac{1}{2}t + \frac{1}{4}e^{-2t} + x_0 \Rightarrow x(0) = 2 = 0 + \frac{1}{4} + x_0$

$\therefore x_0 = 1.75 \qquad \therefore \underline{\underline{x(t) = \frac{1}{2}t + \frac{1}{4}e^{-2t} + \frac{7}{4}}} \qquad (B)$

By Eqn (A); $\quad t = -\frac{1}{2}\ln(1 - 2v) \qquad (C)$

By Eqns (B) and (C):

$x = -\frac{1}{4}\ln(1 - 2v) + \frac{1}{4}e^{\ln(1-2v)} + \frac{7}{4}$

$\therefore \underline{\underline{x(v) = -\frac{1}{4}\left[\ln(1 - 2v) - 8 + 2v\right]}}$

13.26

$a = \sin 2t; \quad v = 0, \; x = 0$ when $t = 0$

$v = \int a\, dt \Rightarrow v = \int \sin 2t\, dt$

$v = -\frac{1}{2}\cos 2t + v_0 \Rightarrow v(t=0) = -\frac{1}{2}(1) + v_0 = 0$

$\therefore v_0 = \frac{1}{2} \qquad \therefore v(t) = \frac{1}{2}(1 - \cos 2t)$

$\underline{\underline{v = \sin^2 t}} \qquad (A)$

$x = \int v\, dt \Rightarrow x = \int\left(\frac{1}{2} - \frac{1}{2}\cos 2t\right) dt$

$x = \frac{1}{2}t - \frac{1}{4}\sin 2t + x_0 \Rightarrow x(0) = 0 - 0 + x_0 = 0$

$\therefore x_0 = 0 \qquad \therefore \underline{\underline{x = \frac{1}{2}t - \frac{1}{4}\sin 2t}} \qquad (B)$

By Eqn (A), $\quad t = \sin^{-1}\sqrt{v} \qquad (C)$

By Eqns (B) + (C),

$\underline{\underline{x(v) = \frac{1}{2}\sin^{-1}\sqrt{v} - \frac{1}{4}\sin\left[2\left(\sin^{-1}\sqrt{v}\right)\right]}}$

13.27

$a = 32 - 4v; \qquad v = 4, \; x = 0$ when $t = 0$

$\int_0^t dt = \int_{v_0}^{v} \frac{dv}{a(v)} \Rightarrow \int_0^t dt = \int_4^v \frac{dv}{32 - 4v}$

Using Integration tables: $\int \frac{dx}{a + bx} = \frac{1}{b}\ln(a + bx)$

$t = -\frac{1}{4}\ln(32 - 4v)\Big|_4^v$

$\therefore \underline{\underline{v(t) = 4\left(2 - e^{-4t}\right)}} \qquad (A)$

By Eqn (A): $\quad x = \int v\, dt = 4\int\left(2 - e^{-4t}\right) dt$

or $\quad x = 8t + e^{-4t} + x_0; \qquad x_0 = -1$ since $x = 0, t = 0$

$\therefore \underline{\underline{x = 8t + e^{-4t} - 1}} \qquad (B)$

By Eqn (A): $\quad t = -\frac{1}{4}\ln\left(\frac{8 - v}{4}\right) \qquad (C)$

By Eqns (B) and (C)

$x = 8\left[-\frac{1}{4}\ln\left(\frac{8-v}{4}\right)\right] + e^{\ln\left(\frac{8-v}{4}\right)} - 1$

Simplifying, $\quad \underline{\underline{x(v) = 3.77 - 2\ln(8 - v) - \frac{v}{4}}}$

13.28 $a = -\frac{4}{x^2}$; $v = 4$, $x = 2$ when $t = 0$ $\quad v > 0$

Find $v(x)$:

$$a = \frac{-4}{x^2} = \frac{dv}{dt}\frac{dx}{dx} = v\frac{dv}{dx} \quad \therefore \int_4^v v\,dv = -4\int_2^x \frac{dx}{x^2}$$

$$\frac{1}{2}v^2\Big|_4^v = \frac{4}{x}\Big|_2^x \quad \text{or} \quad \frac{v^2}{2} = \frac{4}{x} + 6$$

$$\therefore v = 2\sqrt{\frac{3x+2}{x}} \quad (A)$$

Find $x(t)$:

$$v = \frac{dx}{dt} = 2\sqrt{\frac{3x+2}{x}} \qquad \therefore \int_2^x \frac{\sqrt{x}\,dx}{2\sqrt{3x+2}} = \int_0^t dt = t \quad (B)$$

By Integral tables: or computer:

$$\int_2^x \frac{\sqrt{x}\,dx}{\sqrt{3x+2}} = \int_2^x \frac{dx}{\sqrt{\frac{2}{x}+3}} = -2\int_1^{2/x}\frac{dy}{y^2\sqrt{y+3}} \quad (C)$$

where $y = 2/x$.

By Integration tables or computer:

$$\int_1^{2/x}\frac{dy}{y^2\sqrt{y+3}} = -\frac{\sqrt{y+3}}{3y}\Big|_1^{2/x} - \frac{1}{6}\int_1^{2/x}\frac{dy}{y\sqrt{y+3}}$$

$$= -\frac{\sqrt{y+3}}{3y}\Big|_1^{2/x} - \frac{1}{6}\left[\frac{1}{\sqrt{3}}\ln\frac{\sqrt{y+3}-\sqrt{3}}{\sqrt{y+3}+\sqrt{3}}\right]_1^{2/x} \quad (D)$$

By Eqns (B), (C), and (D):

$$t = \frac{1}{6}\sqrt{2x+3x^2} + \frac{1}{10.392}\ln\left[\frac{\sqrt{2x+3x^2}-\sqrt{3}\,x}{\sqrt{2x+3x^2}+\sqrt{3}\,x}\right] - 0.413$$

Find $v(t)$:

By Eqn (A): $x = \frac{8}{v^2-12}$ $\quad$ (F)

Hence, $\sqrt{2x+3x^2} = \frac{4v}{v^2-12}$ $\quad$ (G)

So, by substituting (F), (G), and $x(t)$:

$$t = \frac{2v}{3v^2-36} + \frac{1}{10.392}\ln\left(\frac{v-2\sqrt{3}}{v+2\sqrt{3}}\right) - 0.413$$

for $v > 2\sqrt{3}$

13.29 $a = -8x$; $v = 0$, $x = 6$ when $t = 0$

Find $v(x)$:

$$\tfrac{1}{2}\left(v(x)^2 - v(x_0)^2\right) = \int_{x_0}^{x} a(x)\,dx$$

$$\tfrac{1}{2}\left(v(x)^2 - (0)^2\right) = \int_6^x (-8x)\,dx$$

$$\tfrac{1}{2}v^2 = -4x^2\Big|_6^x \quad \rightarrow \quad v^2 = 288 - 8x^2 \quad (A)$$

$$\therefore v = \sqrt{288-8x^2} \quad \text{or} \quad v = 2\sqrt{72-2x^2}$$

Find $x(t)$:

$$t - t_0 = \int_{x_0}^{x}\frac{dx}{v(x)} \quad \rightarrow \quad t = \int_6^x \frac{dx}{\sqrt{288-8x^2}} + \cancel{t_0}^{\,0}$$

By integration tables:

$$\int\frac{dx}{\sqrt{cx^2+bx+a}} = \frac{1}{\sqrt{-c}}\sin^{-1}\left(\frac{-2cx-b}{\sqrt{b^2-4ac}}\right) \quad \text{for } c < 0$$

Then, $t = \frac{1}{\sqrt{8}}\sin^{-1}\left(\frac{16x}{96}\right) - \frac{1}{\sqrt{8}}\sin^{-1}\left(\frac{16(6)}{96}\right)$

$$t = \frac{1}{\sqrt{8}}\sin^{-1}\left(\frac{x}{6}\right) - \frac{\pi}{2\sqrt{8}}$$

$$t + \frac{\pi}{2\sqrt{8}} = \frac{1}{\sqrt{8}}\sin^{-1}\left(\frac{x}{6}\right) \rightarrow \sin\left(\sqrt{8}\,t + \frac{\pi}{2}\right) = \frac{x}{6}$$

$$\therefore x = \cos(\sqrt{8}\,t)$$

Find $v(t)$:

From Eqn (A): $v^2 = 288 - 8x^2$

and $x = 6\cos(\sqrt{8}t)$ so,

$$v^2 = 288 - 8\left(6\cos(\sqrt{8}t)\right)^2$$

$$= 288 - 288\cos^2(\sqrt{8}t) \qquad = 288(1-\cos^2\sqrt{8}t)$$

$$v^2 = 288\sin^2(\sqrt{8}t) \qquad \therefore v = 12\sqrt{2}\sin(\sqrt{8}t)$$

13.30 $a = -kv$; $v = v_0$, $x = 0$ when $t = 0$

Find $v(t)$:

$$a = -kv = \frac{dv}{dt} \rightarrow \int_0^t dt = -\frac{1}{k}\int_{v_0}^{v}\frac{dv}{v}$$

$$t = -\frac{1}{k}\ln(v)\Big|_{v_0}^{v} = -\frac{1}{k}\ln\frac{v}{v_0}$$

$$\therefore v = v_0 e^{-kt} \quad (A)$$

Find $x(t)$:

$$\frac{dx}{dt} = v \rightarrow \int_0^x dx = \int v\,dt = v_0\int_0^t e^{-kt}\,dt$$

$$x = \frac{-v_0}{k}e^{-kt}\Big|_0^t \quad \text{or} \quad x = \frac{v_0}{k}(1-e^{-kt}) \quad (B)$$

Find $v(x)$:

By Eqn's (A) + (B): $x = \frac{v_0}{k}\left(1-\frac{v}{v_0}\right) = \frac{1}{k}(v_0 - v)$

$$v = v_0 - kx$$

13.31 $a = -kv^2$; $v = v_0$, $x = x_0$ when $t = 0$

Find $v(t)$:

$$a = -kv^2 = \frac{dv}{dt} \rightarrow \int_0^t dt = -\frac{1}{k}\int_{v_0}^{v}\frac{dv}{v^2}$$

$$t = \frac{1}{kv} - \frac{1}{kv_0} \qquad \therefore v = \frac{v_0}{1+kv_0t} \quad (A)$$

Find $x(t)$:

$$v = \frac{dx}{dt} \quad \text{or} \quad \int_0^x dx = \int_0^t v\,dt = \int_0^t \frac{v_0\,dt}{1+kv_0t}$$

$$\therefore x = v_0\int_0^t \frac{dt}{1+kv_0t} = \frac{1}{k}\ln(1+kv_0t) \quad (B)$$

By Eq'ns (A) and (B):

$$x = \frac{1}{k}\ln\frac{v_0}{v}$$

13.32

GIVEN: $Q\,[\text{in}^3/\text{s}]$ = constant

Volume of bubble $= \frac{4}{3}\pi r^3$

FIND $v(r)$ and $a(r)$ for particle of soap film

SOLUTION: Time rate of change of Volume is:

$$Q = \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi r^2 v$$

where $\frac{dr}{dt} = v$ is the velocity of a particle.

Hence, $\underline{\underline{v = \frac{Q}{4\pi r^2}}}$

The acceleration of a particle is given by the chain rule;

$$a = \frac{dv}{dt} = \frac{dv}{dr}\frac{dr}{dt} = v\frac{dv}{dr}$$

$$= \left(\frac{Q}{4\pi r^2}\right)\left(-\frac{2Q}{4\pi r^3}\right) \quad \text{or} \quad \underline{\underline{a = -\frac{Q^2}{8\pi^2 r^5}}}$$

13.33

GIVEN

Position of part given by:

$x = t^4 - 10t^2 + 24$ [mm]

FIND a.) v and a of part

b.) x as function of a

SOLUTION a.) $v(t) = \frac{dx}{dt} = \underline{\underline{4t^3 - 20t}}$ [mm/s]

$a(t) = \frac{d^2x}{dt^2} = \underline{\underline{12t^2 - 20}}$ [mm/s²]

b.) $t(a) = \sqrt{\frac{a+20}{12}}$ → Sub into given $x(t)$

$$\therefore \underline{\underline{x(a) = \left(\frac{a+20}{12}\right)^2 - 10\left(\frac{a+20}{12}\right) + 24}}$$

13.34

GIVEN Body falls 16 ft in first second and falls 32 ft more each second thereafter.

FIND Show this is correct if $g = 32$ ft/s²

SOLUTION

For the first second with $x_0 = v_0 = 0$ for $t_0 = 0$, the body falls a distance:

$$x_1 = 16\text{ ft} = \tfrac{1}{2}at^2 = \tfrac{1}{2}a(1)^2 \quad \therefore \underline{\underline{a = 32 \text{ ft/s}^2}}$$

At end of first second, $v_1 = at = 32(1) = 32$ ft/s

Therefore, at the end of the next second, the body falls an additional distance:

$$x_2 = v_1 t_2 + \tfrac{1}{2}at_2^2 = (32)(1) + \tfrac{1}{2}(32)(1) = 48 \text{ ft}$$

Since 48 ft = 32 ft + 16 ft, this verifies that in the second second, the body falls 32 ft more than it fell in the first second (16 ft).

At the end of the 2^{nd} second,

$$v_2 = v_1 + at = 32 + 32(1) = 64 \text{ ft/s}$$

So, at the end of the third second, the body falls an additional distance:

$$x_3 = v_2 t + \tfrac{1}{2}at_3^2 = (64)(1) + \tfrac{1}{2}(32)(1)^2 = 80 \text{ ft}$$

It can be seen that 80 ft = 48 + 32.

Thus, in the third second, the body falls 32 ft more than it fell (48 ft) in the second second, and so on.

13.35

GIVEN $a = 1$ m/s², $v_{max} = 100$ km/hr

FIND a.) least time [s] required for car to travel 1 km if starting from rest

b.) sketch the velocity-time graph

SOLUTION $v = 100$ km/hr $= 27.78$ m/s $\quad v_0 = 0$

a.) The minimum time to travel 1 km is attained if the car accelerates to 100 km/hr and then maintains that speed.

The time required to attain a speed of 100 km/hr is found by: $v = at$; $\quad 27.78$ m/s $= (1)t$ or $t_1 = 27.78$ s

In this time, the car travels the distance

$$x = \tfrac{1}{2}at^2 = \tfrac{1}{2}(1)(27.78)^2 = 385.8 \text{ m} = 0.386 \text{ km}.$$

In other words, it has not yet traveled 1 km. To travel the remaining distance, the time required is found from the condition that:

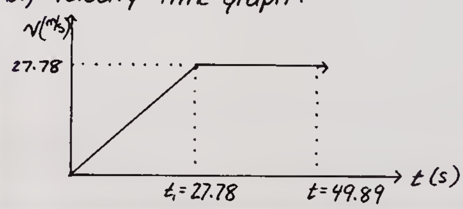
$$\Delta s = (1000 - 385.8) = vt_2 = 27.78\,t_2$$

or $t_2 = 22.11$ s

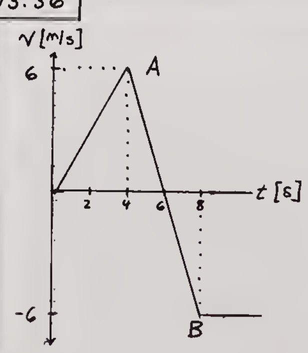
Therefore, total time is:

$$t = t_1 + t_2 = 27.78 + 22.11 = \underline{\underline{49.89 \text{ s}}}$$

b.) Velocity-time graph:



13.36



GIVEN Machine part moves along straight line, has time-velocity graph shown.

FIND a.) displacement of part in intervals $t = 0$–4 s, $t = 4$–8 s, and $t = 0$–8 s.

b.) distance traveled by particle in same intervals

(Continued)

13.36 cont.

SOLUTION a) Displacement, $t=0$ to $4s$:

$\Delta X_{0-4} = \int_0^4 3/2\, t\, dt = 3/4\, t^2 \Big|_0^4 \quad \underline{\underline{\Delta X_{0-4} = 12\,m}}$

Displacement, $t=4$ to $8s$:

$\Delta X_{4-8} = \int_4^8 (-3t+18)\,dt = (-3/2\, t^2 + 18t)\Big|_4^8$

$\underline{\underline{\Delta X_{4-8} = 0\,m}}$

Displacement, $t=0$ to $8s$:

$\Delta X_{0-8} = \int_0^4 3/2\, t\, dt + \int_4^8 (-3t+18)\,dt$

$\underline{\underline{\Delta X_{0-8} = 12\,m}}$

b) Distance traveled, $t=0$ to $4s$:

$\Delta S_{0-4} = |\Delta X_1| + |\Delta X_2| = |0| + |12| \quad \underline{\underline{\Delta S_{0-4} = 12\,m}}$

Distance traveled, $t=4$ to $8s$:

$\Delta S_{4-8} = |\Delta X_1| + |\Delta X_2| = |18-12| + |12-18| = 6+6$

$\underline{\underline{\Delta S_{4-8} = 12\,m}}$

Distance traveled, $t=0$ to $8s$:

$\Delta S_{0-8} = |\Delta X_1| + |\Delta X_2| + |\Delta X_3| = |12-0| + |18-12| + |12-18|$

$= 12 + 6 + 6 \quad \underline{\underline{\Delta S_{0-8} = 24\,m}}$

★ ALTERNATIVE SOLUTION

Displacement from $t=0$ to $t=4s$ is equal to the area of the v-t diagram. Therefore,

$\Delta X_{0-4} = 1/2\,(4)(6) = \underline{\underline{12\,m}}$

Displacement from $t=4$ to $t=8s$:

$\Delta X_{4-8} = 1/2\,(6-4)(6) - 1/2\,(8-6)(6) = \underline{\underline{0\,m}}$

$\therefore$ Displacement from $t=0$ to $t=8s$ is:

$\Delta X_{0-8} = \Delta X_{0-4} + \Delta X_{4-8} = 12 + 0 = \underline{\underline{12m}}$

Distance traveled from $t=0$ to $t=4s$ is

$\Delta S_{0-4} = 1/2\,(4)(6) = \underline{\underline{12m}}$

Distance traveled from $t=4$ to $t=8s$ is

$\Delta S_{4-8} = |\Delta X_{4-6}| + |\Delta X_{6-8}| = |1/2\,(6-4)(6)| + |1/2\,(8-6)(6)|$

$\therefore \Delta S_{4-8} = 6 + 6 = \underline{\underline{12\,m}}$

Distance traveled from $t=0$ to $t=8s$ is

$\Delta S_{0-8} = \Delta S_{0-4} + \Delta S_{4-8}$

$= 12m + 12m \quad \therefore \underline{\underline{\Delta S_{0-8} = 24m}}$

13.37 GIVEN Figure from Problem 13.36

FIND a.) acceleration of part at $t=2s$ + $t=6s$

b.) Discuss Significance of pts A and B

SOLUTION

a.) From v-t diagram:

$$v = \begin{cases} 3/2\, t & 0 \le t < 4 \\ -3t+18 & 4 \le t < 8 \\ -6 & t > 8 \end{cases}$$

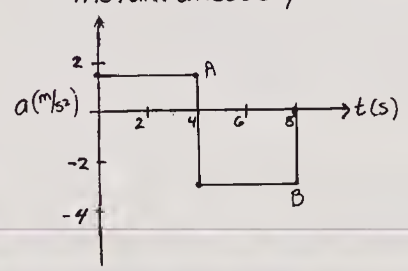
$$a = \frac{dv}{dt}$$

$$a = \begin{cases} 3/2 & 0 \le t < 4 \\ -3 & 4 \le t < 8 \\ 0 & t > 8 \end{cases}$$

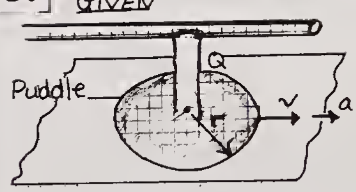
At $t=2s$, $\underline{\underline{a = 1.5\ m/s^2}}$

At $t=6s$, $\underline{\underline{a = -3\ m/s^2}}$

b.) By plotting an a-t diagram, it can be seen that points A and B are points at which the acceleration changes from 1.5 to $-3\ m/s^2$ and from -3 to $0\ m/s^2$ instantaneously.



13.38 GIVEN



h = thickness of puddle = constant

Q = rate at which water leaks [in³/s]

FIND velocity, v and acceleration, a on edge of puddle as functions of r, Q and h.

SOLUTION

Volume of puddle $\Rightarrow V = \pi r^2 h$

By chain rule, $\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt} = 2\pi r h \frac{dr}{dt} = Q$

$$\therefore v = \frac{dr}{dt} = \underline{\underline{\frac{Q}{2\pi r h}}}$$

Again, by chain rule:

$$a = \frac{dv}{dt} = \frac{dv}{dr}\frac{dr}{dt} = \left(-\frac{Q}{2\pi r^2 h}\right)v = \underline{\underline{\frac{-Q^2}{4\pi^2 h^2 r^3}}}$$

13.39

GIVEN Position of body given by:

$x = 3t^2 - 6t \qquad x\,[m],\ t\,[s] \qquad (A)$

FIND a.) $v(t)$

b.) Plot of time - velocity graph

c.) Body's displacement for $t=0$ to $t=2s$ and $t=0$ to $t=4s$

d.) Distance traveled for same intervals

SOLUTION

a.) $v = \frac{dx}{dt} = \underline{\underline{6t - 6\ [m/s]}} \qquad (B)$

b.) By Eq (B), the graph of v vs. t is shown in Figure a:

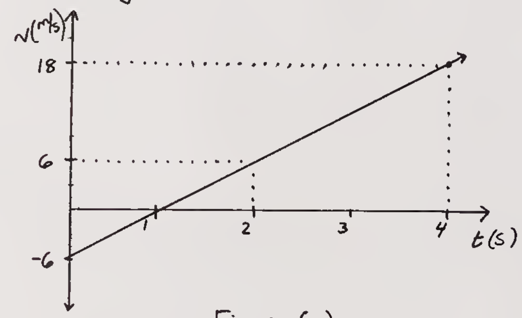


Figure (a)

c.) Displacement: $\Delta x = x_{(final)} - x_{(initial)}$

For $t=0$ to $t=2s$, the displacement is:

$\Delta x_{0\text{-}2} = x(2) - x(0) \qquad (C)$

By Eq (A)

$x(0) = 3(0)^2 - 6(0) = 0$

$x(2) = 3(2)^2 - 6(2) = 0$

$\therefore \underline{\underline{\Delta x_{0\text{-}2} = x(2) - x(0) = 0}}$

For $t=0$ to $t=4s$,

$\Delta x_{0\text{-}4} = x(4) - x(0)$

$x(4) = 3(4)^2 - 6(4) = 24\ m$

$\therefore \underline{\underline{\Delta x_{0\text{-}4} = 24 - 0 = 24\,m}}$

d.) Distance traveled: $\Delta s = |\Delta x_1| + |\Delta x_2|$

For $t=0$ to $t=2s$, noting by Figure a. that $v=0$ at $t=1s$:

$\Delta S_{0\text{-}2} = |\Delta x_{0\text{-}1}| + |\Delta x_{1\text{-}2}|$

By Eq (A):

$|\Delta x_{0\text{-}1}| = |x(1) - x(0)| = |-3-0| = 3m$

$|\Delta x_{1\text{-}2}| = |x(2) - x(1)| = |0+3| = 3m$

$\therefore \underline{\underline{\Delta S_{0\text{-}2} = 3+3 = 6\ m}}$

For $t=0$ to $t=4s$;

$\Delta S_{0\text{-}4} = |\Delta x_{0\text{-}1}| + |\Delta x_{1\text{-}4}|$

By Eq (A), and since v is positive from $t=1$ to $t=4s$,

$|\Delta x_{1\text{-}4}| = |x_4 - x_1| = |24 - (-3)| = 27\ m$

$\therefore \underline{\underline{\Delta S_{0\text{-}4} = 3 + 27 = 30\ m}}$

ALTERNATIVELY:

By Figure (a.)

$\Delta x_{0\text{-}2} = \frac{1}{2}(-6)(1) + \frac{1}{2}(6)(2-1) = 0$

$\Delta x_{0\text{-}4} = \frac{1}{2}(-6)(1) + \frac{1}{2}(18)(4-1) = 24\ m$

$\Delta S_{0\text{-}2} = |\frac{1}{2}(-6)(1)| + |\frac{1}{2}(6)(2-1)| = 6\ m$

$\Delta S_{0\text{-}4} = |\frac{1}{2}(-6)(1)| + |\frac{1}{2}(18)(4-1)| = 30\,m$

13.40

GIVEN First ball thrown upward with velocity v_0. Second ball thrown T-seconds later with same velocity.

FIND

Derive a formula for time t after the second ball is thrown at which the balls pass each other.

SOLUTION

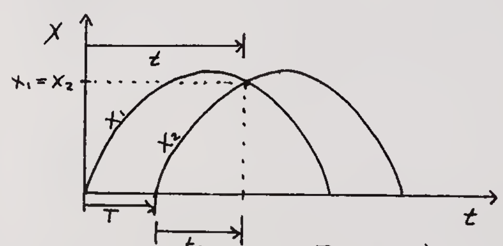


Figure (a.)

Ball one

$a = -g \qquad v_1 = \int a\,dt = -gt + v_0$

$x_1 = \int v_1\,dt = -\frac{1}{2}gt^2 + v_0 t + \cancel{x_0}^{\,0}$

$x_1 = -\frac{1}{2}gt^2 + v_0 t \qquad (A)$

Ball two - thrown T seconds later, $\therefore t_2 = (t - T)$ (See Fig. (a.))

$a = -g \qquad v_2 = \int a\,dt = -gt_2 + v_0$

$x_2 = \int v_2\,dt = -\frac{1}{2}gt_2^2 + v_0 t_2 + \cancel{x_0}^{\,0}$

$x_2 = -\frac{1}{2}gt_2^2 + v_0 t_2 \qquad = -\frac{1}{2}g(t-T)^2 + v_0(t-T) \quad (B)$

Balls pass each other when $x_2 = x_1$ (See Fig (a.))

Equating (A) + (B) gives:

$-\frac{1}{2}gt^2 + v_0 t = -\frac{1}{2}g(t-T)^2 + v_0(t-T)$

Simplifying gives:

$0 = gTt - \frac{1}{2}gT^2 - v_0 T \quad$ or $\quad t = \frac{1}{2}T + \frac{v_0}{g}$

and $t_2 = t - T$

$\therefore t_2 = \frac{1}{2}T + \frac{v_0}{g} - T \qquad \underline{\underline{t_2 = \frac{v_0}{g} - \frac{1}{2}T}}$

13.41 GIVEN Time-velocity graph of racing car as shown (Fig (a))
Note 1in = 10s on time scale,
1in = 5 ft/s on velocity scale

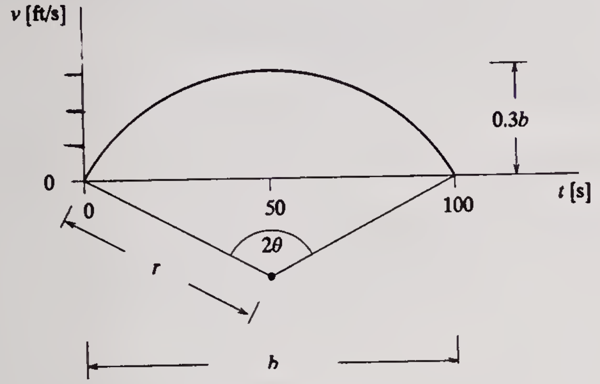
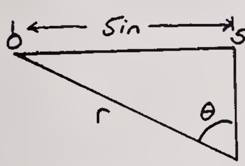


Figure (a)

FIND Distance traveled by car from $t=0$ to $t=100\,s$.

SOLUTION
$b = 100\,s\left(\frac{1\,in}{10\,s}\right) = 10\,in$
$0.3b = 0.3(10\,in) = 3\,in$
$v_{max} = 3\,in\left(\frac{5\,ft/s}{in}\right) = 15\,ft/s$



$x = r - 0.3b = (r-3)\,in$

Figure (b.)

By Fig. (b.):
$r^2 = 5^2 + x^2 = 25 + (r-3)^2$
$r = 17/3\ in$

Also by Fig. (b.):
$5 = r\sin\theta \rightarrow \theta = \sin^{-1}\left(\frac{5}{17/3}\right)$
$\theta = 61.93° = 1.0808\ rad$

By Fig. (a.), the area of the circular section is: $A_{cs} = \theta r^2$
$A_{cs} = (1.0808)(17/3)^2 = 34.706\ in^2$

The area of the triangle is:
$A_{Tri} = 2\left[\tfrac{1}{2}(5in)\left(\tfrac{17}{3} - 3\right)\right] = 13.333\ in^2$

The net area between the t-axis and arc is:
$A_{Net} = A_{cs} - A_{Tri} = 34.706 - 13.333 = 21.373\,in^2$

Therefore, the distance traveled by the car is:
$S = (21.373\,in^2)\left(50\frac{ft}{in^2}\right) = \underline{\underline{1069\ ft}}$

13.42 GIVEN From the instant that the ball is dropped from rest 3m above the floor to the instant that it rebounds, 2m, the time elapsed is 1.48s.
Also, $g = 9.81\ m/s^2$.

FIND Time [s] that ball remains in contact with the floor.

SOLUTION

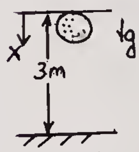


Fig (a.)

Ball is released at time $t=0$ and drops $x=3m$ (Fig (a.)).
With the relationship
$x = \tfrac{1}{2}at^2 + v_0 t + x_0$ (A)
and t_1 = time the instant before the ball touches the ground,
$3 = \frac{9.81}{2}t_1^2 + 0(t_1) + 0 \qquad t_1 = 0.782\,s$

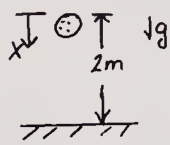


Fig. (b.)

The ball rebounds to a height 2m above the floor (Fig (b.)). Therefore, at $x=2m$ above floor, $v=0$.

The time t_2 that it takes the ball to rise 2m is the same time it takes to fall 2m, starting from rest. ($x_0 = v_0 = 0$). Therefore, by Eq. (A) and Fig (b.):
$2m = \tfrac{1}{2}g t_2^2 = \tfrac{1}{2}(9.81)t_2^2$
OR $t_2 = 0.6385\,s$

Hence, the time that the ball is in contact with the floor is:
$t = 1.48\,s - t_1 - t_2$
$= 1.48\,s - 0.7821\,s - 0.6385\,s$
$\therefore \underline{\underline{t = 0.059\,s}}$

13.43 GIVEN $a = -3t\ [m/s^2]$
at $t=0$, $x = 6m\ (=x_0)$ $v = 4\,m/s\ (=v_0)$

FIND a.) x, v, & a at $t=2s$
b.) Displacement and distance traveled in intervals $t=0-2s$ and $t=0-4s$.

SOLUTION $v(t) = \int a\,dt = \int -3t\,dt = -\tfrac{3}{2}t^2 + v_0$
Since $v_0 = 4$, $v(t) = -\tfrac{3}{2}t^2 + 4$ (A)
$x(t) = \int v\,dt = \int(-\tfrac{3}{2}t^2 + 4)dt = -\tfrac{1}{2}t^3 + 4t + x_0$
Since $x_0 = 6m$, $x(t) = -\tfrac{1}{2}t^3 + 4t + 6$ (B)

a.) By Eqs.(B) and (A),
$x(t=2) = \underline{\underline{10m}}$
$v(t=2) = \underline{\underline{-2\,m/s}}$
$a(t=2) = \underline{\underline{-6\,m/s^2}}$

(Continued)

13.43 cont.

b.) Displacement

For $t=0$ to $t=2s$,

$\Delta X_{0-2} = x(2s) - x(0)$, where $x(0) = x_0 = 6m$

By Eq. (B): $x(2s) = -\frac{1}{2}(2)^3 + 4(2) + 6 = 10m$

$\therefore \Delta X_{0-2} = 10 - 6 = 4m$

For $t=0$ to $t=4s$,

$\Delta X_{0-4} = X(4s) - X_0$

By Eq. (B): $X(4s) = -\frac{1}{2}(4)^3 + 4(4) + 6 = -10m$

$\therefore \Delta X_{0-4} = -10 - 6 = -16\,m$

Distances

Note that, by Eq (A), v is positive from $t=0$ to $t=\sqrt{8/3}$ s $= 1.630$s. From $t=1.630$s to $t=2$s, v is negative. Thus,

$\Delta S_{0-2} = |\Delta X_{0-1.630}| + |\Delta X_{1.630-2}|$

where, By Eq (B),

$\Delta X_{0-1.630} = X(1.630\,s) - X(0) = 10.343 - 6 = 4.343$

$\Delta X_{1.630-2} = X(2s) - X(1.630) = 10 - 10.343 = -0.343$

$\therefore \Delta S_{0-2} = |4.343| + |-0.343| = 4.69\,m$

From $t=0$ to $t=4s$, v is negative from $t=1.630$ s to $t=4s$.

$\therefore \Delta S_{0-4} = |\Delta X_{0-1.630}| + |\Delta X_{1.630-4}|$

where $\Delta X_{0-1.630} = 4.343$ m as before and by Eq (B):

$\Delta X_{1.634-4} = X(4s) - X(1.630) = -10 - 10.343 = -20.343$

$\therefore \Delta S_{0-4} = |4.343| + |-20.343| = 24.69\,m$

13.44 Computer Problem

Given

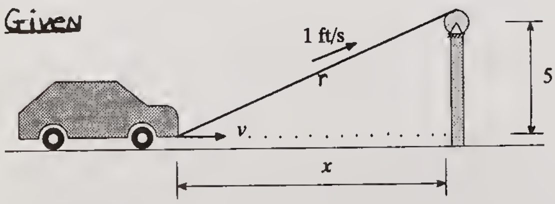


Figure (a)

Find a.) Plot speed, v [ft/s] and acceleration a [ft/s²] of car as function of x for $1 \le x \le 12$ ft.

b.) Is this method of pulling the car a good one? Explain.

Solution

By Fig (a), $r^2 = x^2 + 5^2$

Therefore: $2r\dot{r} = 2x\dot{x}$, or $r v_R = x v$; ($v_R = 1$ ft/s)

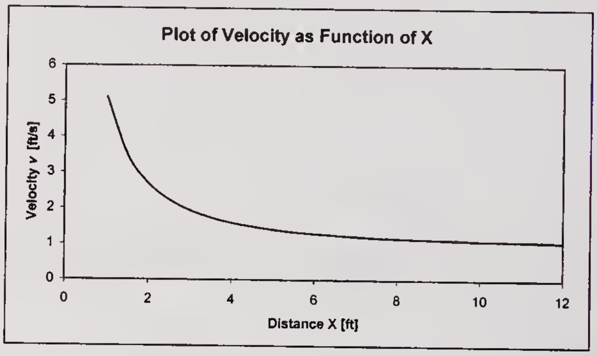
So, $v = \frac{r}{x} = \sqrt{1 + \frac{25}{x^2}}$ [ft/s] (A)

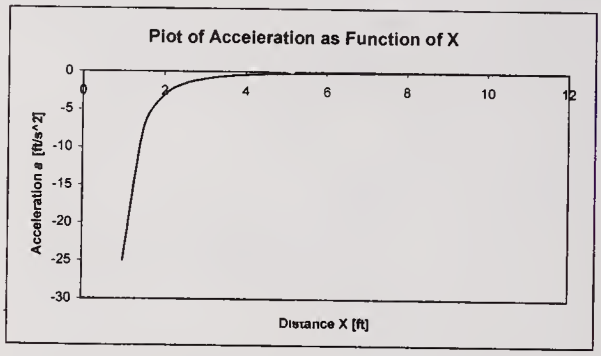
Note that as $x \to 0$, $v \to \infty$ (see plot).

Differentiation of Eq (A) yields the acceleration. By chain rule:

$a = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx} = -\frac{25}{x^3}$ [ft/s²] (B)

Again, note that as $x \to 0$, $a \to \infty$ (see plot).





b.) This method is not a good one, since velocity and acceleration $\to \infty$ as $x \to 0$. $\therefore$ The car runs the risk of running into the pulley pole at such a high speed.

13.45 Computer Problem

Given Jet propelled boat moves in straight line such that position is $X = t^3 + 6t^2 + 5$ [ft] where t denotes time in seconds.

Find a.) Plot curves for position, velocity, and acceleration as functions of t for $0 \le t \le 4s$.

b.) v_{AV} + a_{AV} for $0 \le t \le 4$s and plot on graphs of part a.

c.) Est. time for which $v = v_{AV}$; $a = a_{AV}$.

d.) Verify by calculation.

(Continued)

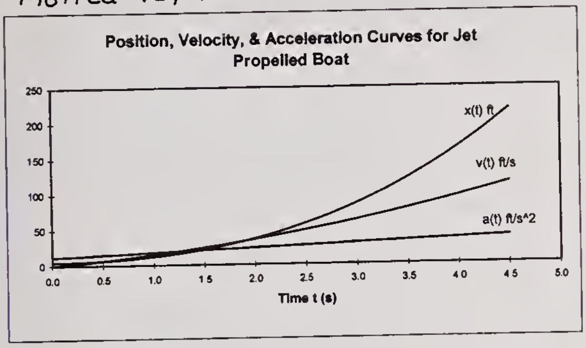
13.45 cont.

SOLUTION $x = t^3 + 6t^2 + 5$ [ft]

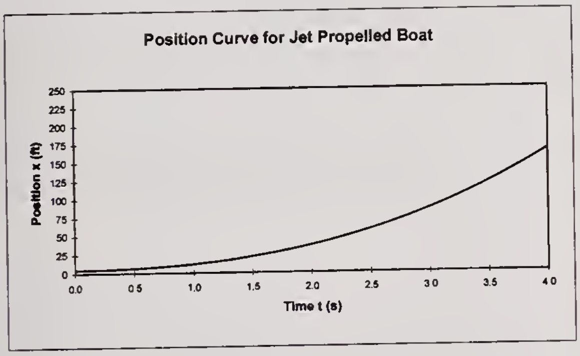
$\therefore v(t) = 3t^2 + 12t$ [ft/s]

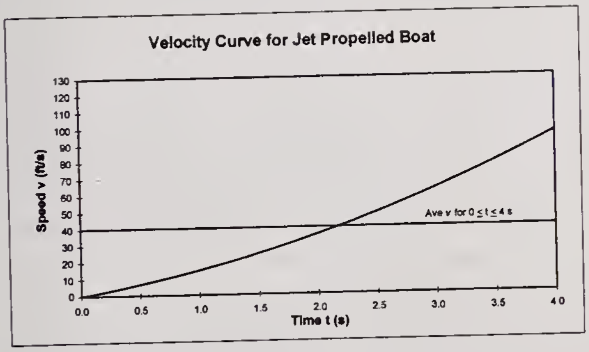
$a(t) = 6t + 12$ [ft/s²]

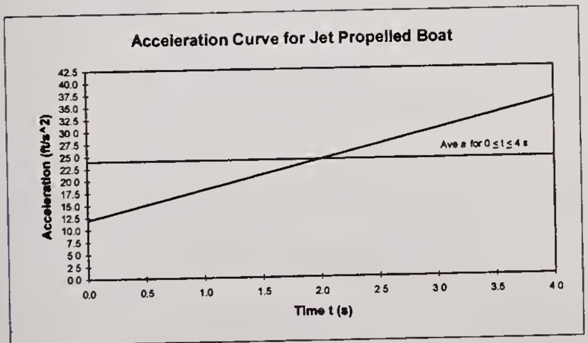
a.) Plotted Together:



Plotted Seperately:







b.) Average speed and acceleration:

$$v_{Ave} = \frac{1}{4}\int_0^4 [3t^2 + 12t]\,dt = \underline{\underline{40 \text{ ft/s}}}$$

→ plotted on v-t graph above

$$a_{ave} = \frac{1}{4}\int_0^4 (6t + 12)\,dt = 24 \text{ ft/s}^2$$

→ plotted on a-t graph above

c.) From plots obtained in part a.) and averages obtained in part b.):

t for instantaneous velocity = ave. velocity $\underline{\underline{\approx 2.15\,s}}$

t for instantaneous acceleration = ave acceleration $\underline{\underline{\approx 2s}}$

d.) These results can be verified by calculation using the average values found in part b.) and solving for t.

$v_{ave} = 40 \text{ ft/s} = 3t^2 + 12t$ $\quad \underline{\underline{t = 2.16s}}$

$a_{Ave} = 24 \text{ ft/s} = 6t + 12$ $\quad \underline{\underline{t = 2s}}$

13.46

GIVEN Particle moves on straight line with velocity shown on time-velocity graph shown in Figure (a.):

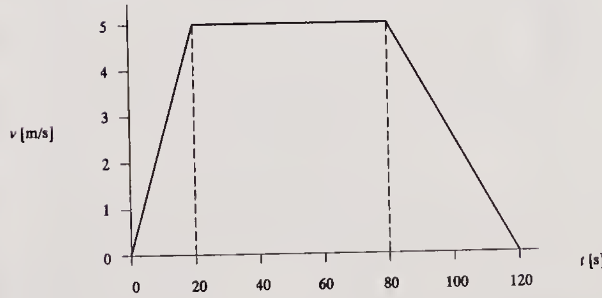


Figure (a)

FIND Plot time-acceleration and time-position graphs and from these plots, determine the acceleration and position for $t = 20s, 60s, 80s,$ and $120s$.

SOLUTION By Figure (a.)

$$v(\text{m/s}) = \begin{cases} \tfrac{1}{4}t, & 0 \le t \le 20s \\ 5, & 20 \le t \le 80s \\ -\tfrac{1}{8}t + 15, & 80 \le t \le 120s \end{cases}$$

Also by Figure (a.), the acceleration is the slope of the velocity curve (i.e. $a = dv/dt$) Therefore,

$$a(\text{m/s}^2) = \begin{cases} \tfrac{1}{4}, & 0 \le t \le 20s \\ 0, & 20 \le t \le 80s \\ -\tfrac{1}{8}, & 80 \le t \le 120s \end{cases}$$

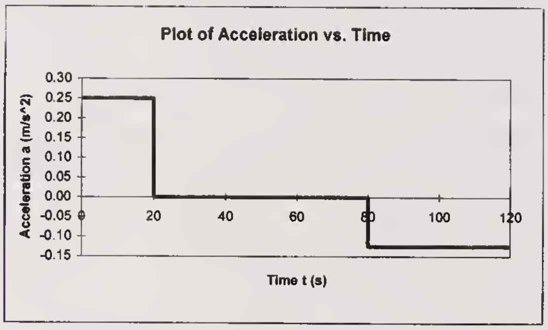
(Continued

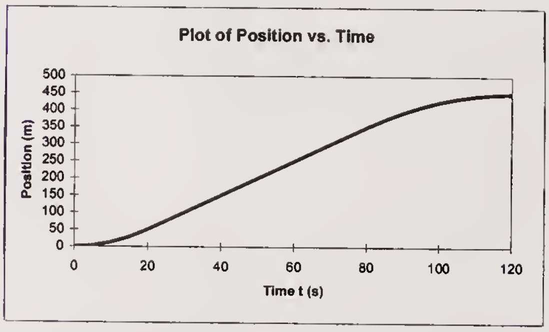
13.46 cont.

The displacement, x, is obtained by integration: i.e. $x=\int v\,dt$. Therefore, with Eq (A),

$$x\,(m)=\begin{cases} \frac{1}{8}t^2 & 0\le t\le 20 \\ 5t-50 & 20\le t\le 80 \\ -\frac{1}{16}t^2+15t-450 & 80\le t\le 120 \end{cases} \qquad (C)$$

With Eqs (B) & (C), the time-acceleration and time-position plots are as follows:





From these plots, it can be determined that:

@ $t=20\,s$: $a=\frac{1}{4}\,m/s^2$, $x=50\,m$

@ $t=60\,s$: $a=0\,m/s^2$ $x=250\,m$

@ $t=80\,s$: $a=-\frac{1}{8}\,m/s^2$ $x=350\,m$

@ $t=120\,s$: $a=-\frac{1}{8}\,m/s^2$ $x=450\,m$

13.47 GIVEN $a=-16x$ (m/s^2) ; when $x=0$, $v=20\,m/s$

FIND a.) x when $v=0$

b.) Plot the geometric path in x, v plane

c.) Determine distance traveled from $x=0$ to position when $v=-10\,m/s$ for first time.

SOLUTION

By the chain rule, $a=\frac{dv}{dt}=\frac{dv}{dx}\frac{dx}{dt}=v\frac{dv}{dx}$

or $\int_0^x a\,dx=\int_0^x(-16x)\,dx=\int_{20}^{v} v\,dv$

Integration yields:

$$-8x^2=\tfrac{1}{2}(v^2-400) \qquad (A)$$

For $v=0$, Eq (A) yields: $\underline{\underline{x=5\,m}}$ (B)

b.) Geometry of Path in (x,v) plane

By Eq (A): $v^2+16x^2=400$ (C)

or $\frac{v^2}{20^2}+\frac{x^2}{5^2}=1$ (D)

Equation (D) is the equation of an ellipse (see Fig. (a.))

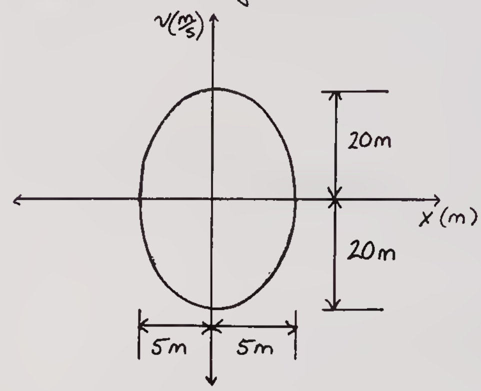


Figure (a.)

c.) The distance traveled by the particle from $x=0$ to x at $v=-10\,m/s$ for first time:

when $x=0$, $v=20\,m/s$ (Given) and,

when $v=0$, $x=5$ (by Eq. (D))

Hence, velocity changes signs (direction) at $x=5\,m$. By Eq. (C):

$$v=\pm\sqrt{400-16x^2} \qquad (E)$$

Thus, for $v=-10\,m$, Eq. (E) yields:

$$-10=-\sqrt{400-16x^2} \qquad \underline{\underline{x=4.33\,m}}$$

So, the distance traveled from $x=0$ to position when $v=-10\,m/s$ is

$$\Delta s=|\Delta x_1|+|\Delta x_2|$$
$$=|5-0|+|4.33-5| \quad \text{or}$$

$$\underline{\underline{\Delta s=5.67\,m}}$$

13.48 GIVEN $a=\frac{x^3}{2}$; when $t=1$, $x=1$, $v=0$ and $v\ge 0$.

FIND $v(x)$ and $t(x)$ and $v(t)$, $x(t)$, $a(t)$

Sketch graphs of all 5.

SOLUTION

By the chain rule, $a=\frac{dv}{dt}=\frac{dv}{dx}\frac{dx}{dt}=v\frac{dv}{dx}$

$\therefore \int_0^v v\,dv=\frac{1}{2}\int_0^x x^3\,dx$

or $\frac{v^2}{2}=\frac{x^4}{8} \Rightarrow v=\pm\frac{x^2}{2}$

Since $v\ge 0$, $\underline{\underline{v=\frac{x^2}{2}}}$ (A)

(Continued)